Computational Voting Theory, Algorithmic Game Theory and the Connections Between Them

Progress Lecture by Michael Zuckerman
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Agenda

- Introduction
- Computational Voting Theory (CVT)
  - Manipulation under Maximin
- Algorithmic Game Theory (AGT)
  - Transformation Games
  - Effort Games
- The connections between CVT and AGT
  - Manipulating the quota in WVG
  - An NTU cooperative game theoretic view of manipulating elections
- Summary

Computational Voting Theory – Introduction

- Voting enables a group of agents (voters) to make a joint choice from a set of alternatives
- Each voter reports his preferences over the alternatives
- A voting rule is applied to aggregate the preferences of the agents

Manipulation in Voting

- Sometimes a subset of the agents can gain by reporting their preferences insincerely
- A voting rule is strategy-proof if there is never a (beneficial) manipulation under this rule.

Gibbard-Satterthwaite Theorem

- When there are at least 3 alternatives, there is no voting rule that satisfies all of the following conditions:
  - Non-imposition: every alternative wins under some profile
  - Non-dictatorship: there is no voter such that we always choose that voter's most preferred alternative
  - Strategy-proof (non-manipulable)

Computational Complexity as a Barrier Against Manipulation

- Second order Copeland and STV are NP-hard to manipulate [Bartholdi et al. 89, Bartholdi & Orlin 91]
- Many hybrids of voting rules are NP-hard to manipulate [Conitzer & Sandholm 03, Elkind and Lipmaa 05]
- Many common voting rules are hard to manipulate for weighted coalitional manipulation [Conitzer et al. 07]
- All of these are *worst-case* results: it could be that *most* instances are easy to manipulate
  - Some evidence that this is indeed the case [Procaccia & Rosenschein 06, Conitzer & Sandholm 06, Zuckerman et al. 09, Friedgut et al. 08, Xia & Conitzer 08a, Xia & Conitzer 08b, Isaksson et al. 10]

Algorithmic Game Theory – Introduction

- Combines computer science, game theory and social choice
- Studies interactions among electronic agents
- The agents are often designed to be:
  - Intelligent
  - Self-interested
  - Computationally efficient

Introduction

- Non-cooperative game theory
  - Individually acting agents
  - Nash equilibrium
  - Dominant-strategy equilibrium
- Cooperative game theory
  - Agents forming coalitions
  - What coalitions are most likely to arise
  - How the gains from cooperation are distributed among the agents

Cooperative Game Theory

- Stability in cooperative games
  - The core
  - ε-core
  - Least-core
- Power indices
  - Shapley-Shubik index
  - Banzhaf index
  - Always exist
  - Unique
  - In general are hard to compute
    - There are efficient approximation algorithms

An Algorithm for the Coalitional Manipulation Problem under Maximin

AAMAS'11

Joint work with Omer Lev and Jeff Rosenschein

Agenda

- Introduction
- Constructive Coalitional Unweighted Manipulation (CCUM) problem
- Algorithm for CCUM under Maximin
- 5/3-approximation of the optimum
- Lower bound on approximation ratio
- Simulation results
- Conclusions

Constructive Coalitional Unweighted Manipulation (CCUM) problem

- Given
  - A voting rule *r*
  - The Profile of Non-Manipulators *PNM*
  - Candidate *p* preferred by the manipulators
  - Number of manipulators $|M|$
- We are asked whether or not there exists a Profile of Manipulators *PM* such that *p* is the winner of *PNM* ∪ *PM* under *r.*

Constructive Coalitional Unweighted Optimization (CCUO) problem

- Given
  - A voting rule *r*
  - The Profile of Non-Manipulators *PNM*
  - Candidate *p* preferred by the manipulators
- We are asked to find the minimum *k* such that there exists a set of manipulators *M* with $|M| = k$, and a Profile of Manipulators *PM* such that *p* is the winner of $PNM \cup PM$ under *r.*

Our setting, Maximin

- $C = \{c_1,\ldots,c_m\}$ – the set of candidates
- S, $|S| = N$ – the set of N non-manipulators
- T, $|T| = n$ – the set of n manipulators
- $N_i(c, c') = |\{ k \mid c >_k c', >_k \in S \cup \{1,\ldots,i\}\}|$ – the number of voters from S and from the i first manipulators, which prefer c over c'
- $S_i(c) = \min_{c' \neq c} N_i(c, c')$ – the Maximin score of c from S and the i first manipulators
- Maximin winner $= \mathrm{argmax}_c\{S_n(c)\}$
- Denote $MIN_i(c) = \{c' \in C \mid S_i(c) = N_i(c, c')\}$

Maximin example


a > b > c > d


b > d > a > c


c > b > d > a

	a	b	c	d	score
a	-	1	2	1	1
b	2	-	2	3	2
c	1	1	-	2	1
d	2	0	1	-	0

CCUM Complexity

- CCUM under Maximin is NP-complete for any fixed number of manipulators (≥ 2)

  (Xia et al. ’09 [7])
- It follows that the CCUO is not approximable by constant better than 3/2, unless P = NP
  - Otherwise, if opt = 2, then the output of the algorithm would be < 3, i.e., 2
  - Hence, it would solve the CCUM for n = 2, a contradiction

The heuristic / approximation algorithm

- Fix some order on manipulators
- The current manipulator i
  - Ranks p first
  - Builds a digraph $G^{i-1} = (V, E^{i-1})$, where
    - $V = C \setminus \{p\}$;

      $(x, y) \in E^{i-1}$ iff $(y \in MIN_{i-1}(x)$ and $p \notin MIN_{i-1}(x))$
  - Iterates over the candidates who have not yet been ranked
  - If there is a candidate with out-degree 0, then it adds such a candidate with the lowest score
  - Otherwise, adds a vertex with the lowest score
  - Removes all the outgoing edges of vertices who had outgoing edge to newly added vertex

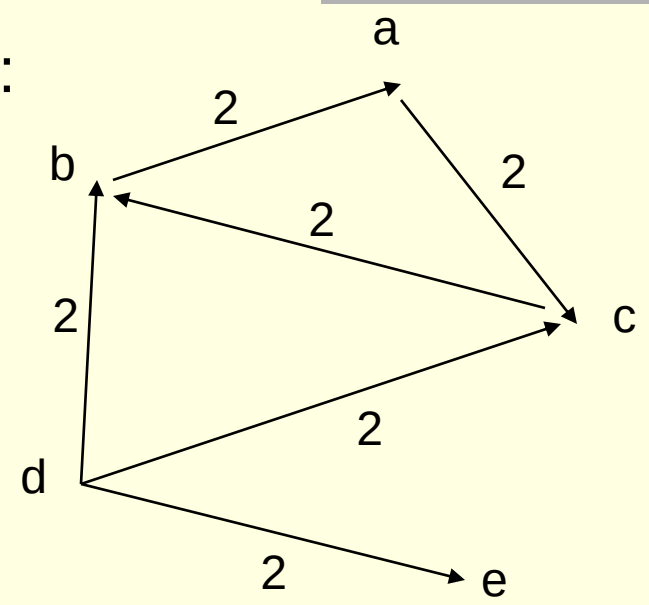
Additions to the algorithm

- The candidates with out-degree 0 are kept in stacks in order to guarantee a DFS-like order among the candidates with the same scores
- If there is no candidate (vertex) with out-degree 0, then it first searches for a cycle, with 2 adjacent vertices having the lowest scores
  - If it finds such a pair of vertices, it adds the front vertex

Example – 1/4

- C = {a, b, c, d, e, p}
- |S| = 6
- |T| = 2
- The non-manipulators' votes:
  - a > b > c > d > p > e
  - a > b > c > d > p > e
  - b > c > a > p > e > d
  - b > c > p > e > d > a
  - e > d > p > c > a > b
  - e > d > p > c > a > b

G^0:



$S_0(p) = N_0(p, b) = 2$

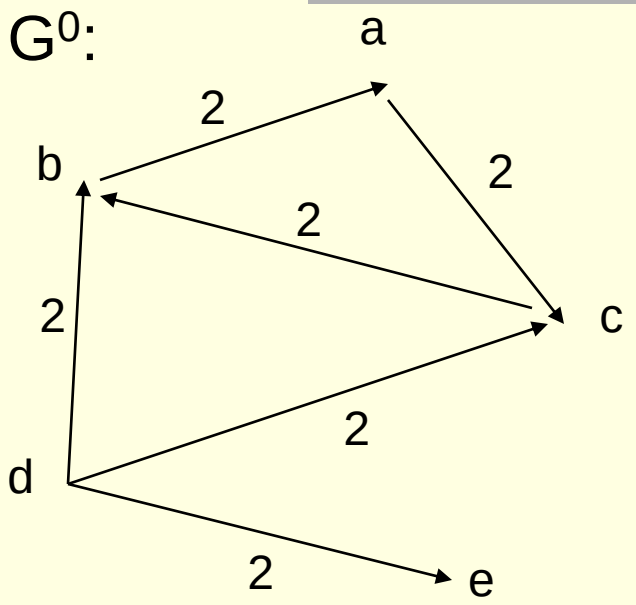
$S_0(e) = N_0(e, p) = 2$

Example - 2/4

- The non-manipulators' votes:
  - a > b > c > d > p > e
  - a > b > c > d > p > e
  - b > c > a > p > e > d
  - b > c > p > e > d > a
  - e > d > p > c > a > b
  - e > d > p > c > a > b
- The manipulators' votes:

p > e > d > b > c > a

G^0:



$S_0(p) = N_0(p, b) = 2$

$S_0(e) = N_0(e, p) = 2$

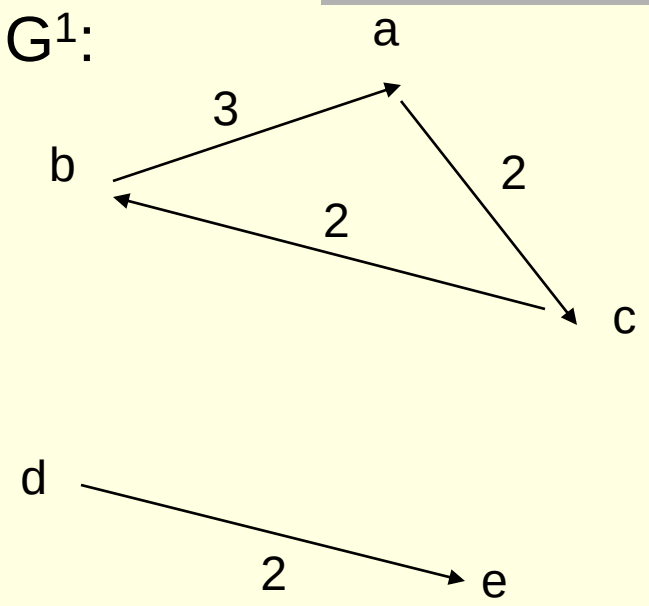
Example - 3/4

- The non-manipulators' votes:
  - a > b > c > d > p > e
  - a > b > c > d > p > e
  - b > c > a > p > e > d
  - b > c > p > e > d > a
  - e > d > p > c > a > b
  - e > d > p > c > a > b
- The manipulators' votes:

p > e > d > b > c > a

p > e > d > c > a > b

G^1:



$S_1(p) = N_1(p, b) = 3$

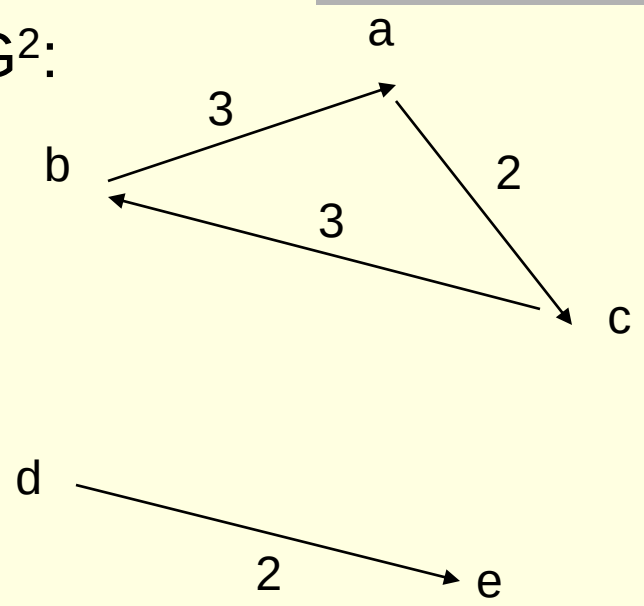
$S_1(e) = N_1(e, p) = 2$

Example - 4/4

- The non-manipulators' votes:
  - a > b > c > d > p > e
  - a > b > c > d > p > e
  - b > c > a > p > e > d
  - b > c > p > e > d > a
  - e > d > p > c > a > b
  - e > d > p > c > a > b
- The manipulators' votes:

  p > e > d > b > c > a
  p > e > d > c > a > b

G^2:



$S_2(p) = N_2(p, b) = 4$
$\max_{x \neq p} S_2(x) = 3$

p is the winner!

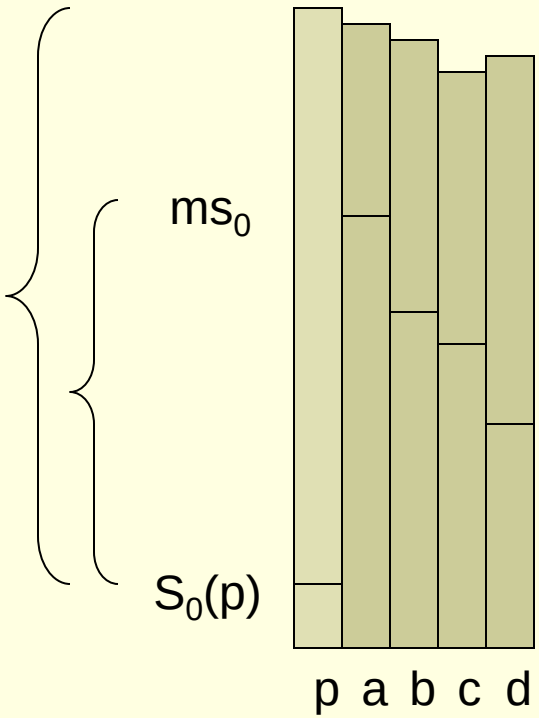
Instances without 2-cycles

- Denote $ms_i = \max_{c \neq p} S_i(c)$
  - The maximum score of *p*'s opponents after *i* stages
- Lemma: If there are no 2-cycles in the graphs built by the algorithm, then for all i, $0 \leq i \leq n-3$ it holds that $ms_{i+3} \leq ms_i + 1$
- Theorem: If there are no 2-cycles, then the algorithm gives a 5/3-approximation of the optimum

Proof of Theorem

$n = \lceil (3\, ms_0 - 3S_0(p) + 3)/2 \rceil$

$opt \geq ms_0 - S_0(p) + 1$

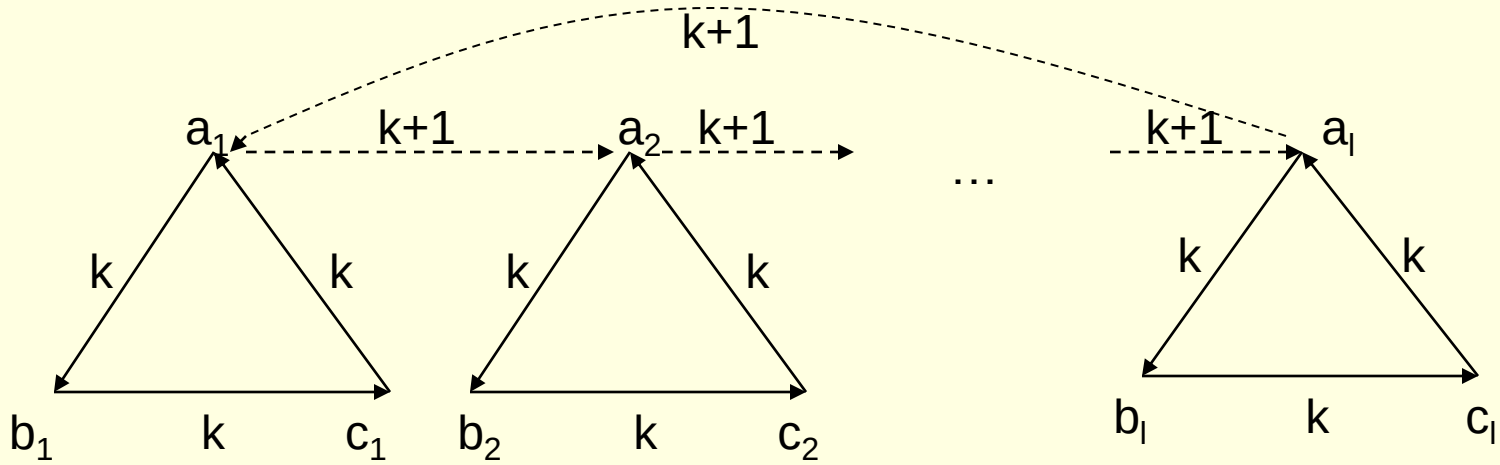


The ratio n/opt is the biggest when opt = 3, $n = \lceil 3/2 * 3 \rceil = 5$

Eliminating the 2-cycles

- Lemma: If at a certain stage i there are no 2-cycles, then for all j > i, there will be no 2-cycles at stage j
- We prove that the algorithm performs optimally while there are 2-cycles
  - Intuitively, if there is a 2-cycle, then one of its vertices has the highest score, and it will always be placed in the end – until the cycle is eliminated
- Once the 2-cycles have been eliminated, our algorithm performs a 5/3-approximation on the number of stages left
- Generally we have 5/3-approximation of the optimal solution

Lower bound on approximation ratio



- When voted:

$p > a_l > c_l > b_l > \ldots > a_1 > c_1 > b_1$

…

- ms^*_i grows by 1 every (m-1)/3 voters

- Our algorithm can vote:

$p > a_1 > c_1 > b_1 > \ldots > a_l > c_l > b_l$

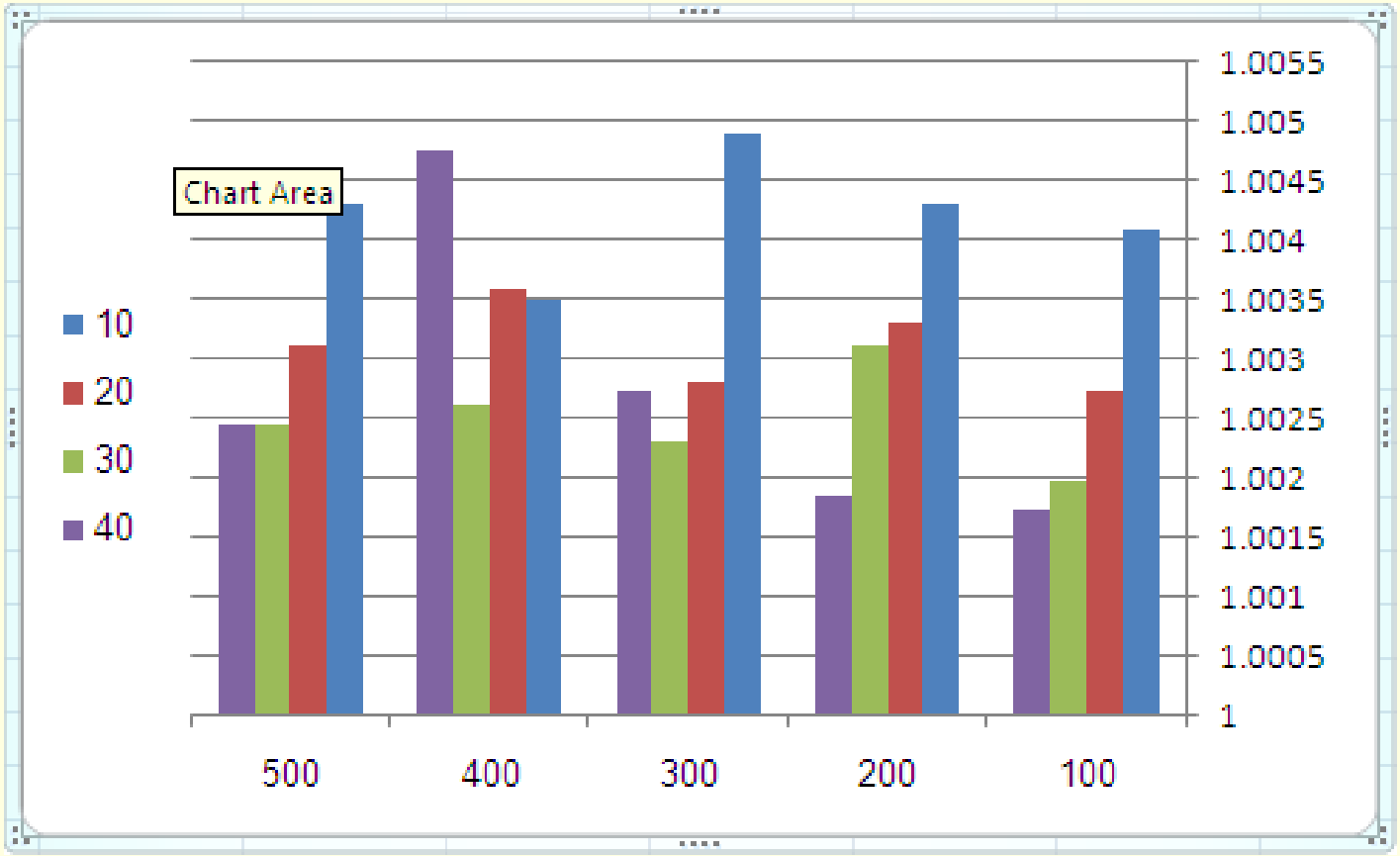
…

- Here ms_i grows by 1 every 3 voters

Simulation results

- Implemented
  - this algorithm
  - the simple greedy algorithm in [8]
- On average, this algorithm is a little better
- On some instances, the simple greedy is better
- No difference between the performance of this algorithm with and without the “additions”
- Difficulties in calculating the optimum

Simple greedy alg / this alg



10 – 40 candidates
100 – 500 non-manipulators

Conclusions

- A new heuristic / approximation algorithm for CCUM / CCUO under Maximin
- Gives a 5/3-approximation of the optimum
- The lower bound on the approximation ratio of the algorithm (and any algorithm) is 1½
- Simulation results – comparison between this algorithm and the simple greedy algorithm in [8]

- Future work
  - Prove the approx. ratio without the additions

Manipulation with Randomized Tie-Breaking under Maximin

AAMAS'12

Joint work with Jeff Rosenschein

The setting

- Dealing with Maximin voting rule
- There is only one manipulator
- Ties are broken uniformly at random
- The manipulator has utilities for the candidates: (1,…,1,0,…,0) with k 1's
- The problem is to maximize the manipulator's expected utility

The results

- The problem is in FPT when parameterized by k
  - Relies on algorithm of Obraztsova et al. (AAMAS'11)
- Under certain conditions it is approximable by a constant factor
- Under certain other conditions it can be solved exactly

Proof Systems and Transformation Games

CoopMAS'10

Joint work with Yoram Bachrach, Michael Wooldridge and Jeff Rosenschein

Transformation Games – 1/2

- Cooperative game
- Players are endowed with
  - initial resources
  - capabilities of deriving certain output resources given certain input resources
- The aim is to generate a particular target resource
- It is achieved by forming coalitions

Transformation Games – 2/2

- Can model cooperative proof systems and supply chains
- We considered possible restrictions on transformation chains
- We considered the computational complexity of:
  - Testing whether a coalition wins
  - Checking whether a player is a dummy or a veto player
  - Computing the core of the game
  - Computing power indices
- Extension: the transformations have associated costs

Effort Games and the Price of Myopia

MLQ’09

Joint work with Yoram Bachrach and Jeff Rosenschein

Effort Games and the Price of Myopia (PoM) – 1/3

- Multi-agent environment
- A common project depends on various tasks
  - Winning and losing subsets of tasks
- The probability of carrying out a task is higher when the agent in charge of it exerts effort
- There is a certain cost for exerting effort
- The principal tries to incentivize agents to exert effort
- The principal can only reward agents based on success of the entire project

Effort Games and the PoM – 2/3

- The PoM is a measure of the influence the model of rationality has on the minimal payments the principal has to make in order to motivate the agents to exert effort.
- We considered the computational complexity of testing whether exerting effort is a dominant strategy for an agent
- The computational complexity of finding a reward strategy, using:
  - Dominant strategy equilibrium, or
  - Iterated elimination of dominated strategies

Effort Games and the PoM - 3/3

- Proved that the above problems are generally #P-hard
  - At least as hard as calculating the Banzhaf power index
- In a certain restricted domain all of the above problems are in P
- Gave bounds on PoM in WV Effort Games
- Provided simulation results in EG played over Series-Parallel Graphs

Manipulating the Quota in Weighted Voting Games

AIJ'12

Joint work with Piotr Faliszewski, Yoram Bachrach and Edith Elkind

Manipulating the Quota in Weighted Voting Games – 1/2

- WVG are described by
  - A set of players
  - A list of players' weights
  - A quota
- The power of a player is identified with
  - His Shapley-Shubik index, or
  - His Banzhaf index

Manipulating the Quota in Weighted Voting Games – 2/2

- Investigated, how much the central authority can change a player's power, by modifying the quota
- Studied, how the choice of quota can affect the relative power of players
- Provided an efficient algorithm for determining whether there is a value of the quota that makes a given player a dummy
- Hardness result for checking which of the two values of the quota makes a player more powerful.

An NTU Cooperative Game Theoretic View of Manipulating Elections

WINE'11

Joint work with Piotr Faliszewski, Vincent Conitzer and Jeff Rosenschein

Constructive Coalitional Unweighted Manipulation (CCUM) Problem

- Given
  - A voting rule *R*
  - The Profile of Non-Manipulators *PNM*
  - Candidate *c* preferred by the manipulators
  - Number of manipulators $|M|$
- We are asked whether or not there exists a Profile of Manipulators *PM* such that *c* is the winner of *PNM* ∪ *PM* under *R.*

Motivation

- In CCUM
  - We assume that the manipulators are single-minded
  - We do not ask how they choose the candidate for whom to vote
- Here
  - We use cooperative game theory to fill this gap
  - Explore the Non-Transferable Utility (NTU) setting

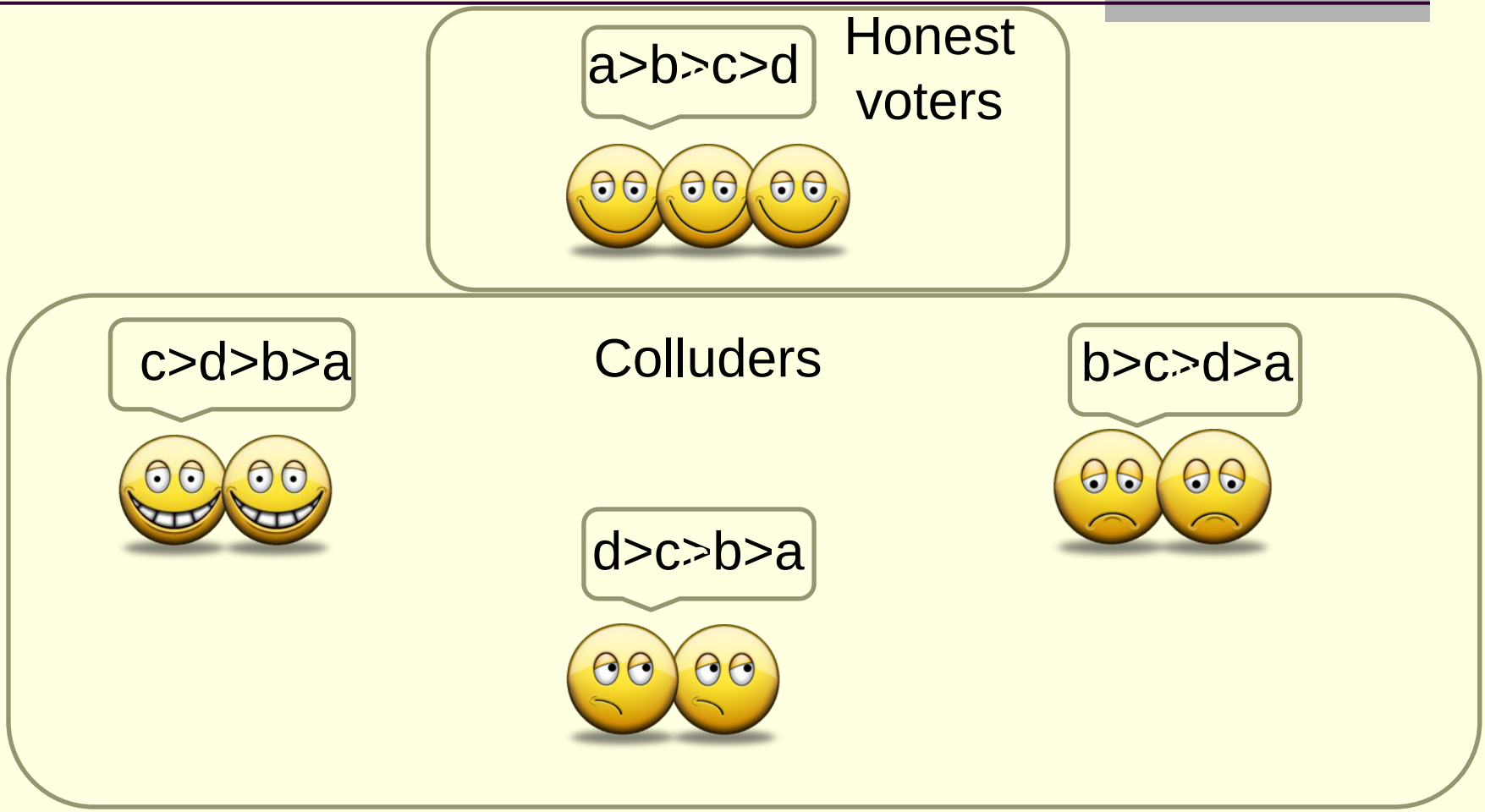
The three settings

- Assume that the manipulators can communicate with one another
- What a (sub)coalition of the manipulators can achieve depends on the votes of manipulators outside the coalition
- Consider 3 settings:
  - Strong Nash equilibrium
  - Pessimistic (α-core)
  - Adaptive (β-core)

Notation

- $C = \{c_1, \ldots, c_m\}$ – the candidates/alternatives
- $V = \{1, 2, \ldots, n\}$ – the voters
- $P = \{P_1, \ldots, P_n\}$ – a preference profile of the voters
- $R(P)$ – the winner under profile P
- $M \subseteq V$ – the colluders/manipulators
- $H = V \setminus M$ – the honest voters
- $W_{ab} \subseteq M$ – all the colluders who prefer a to b

Example



The truthful plurality winner is a

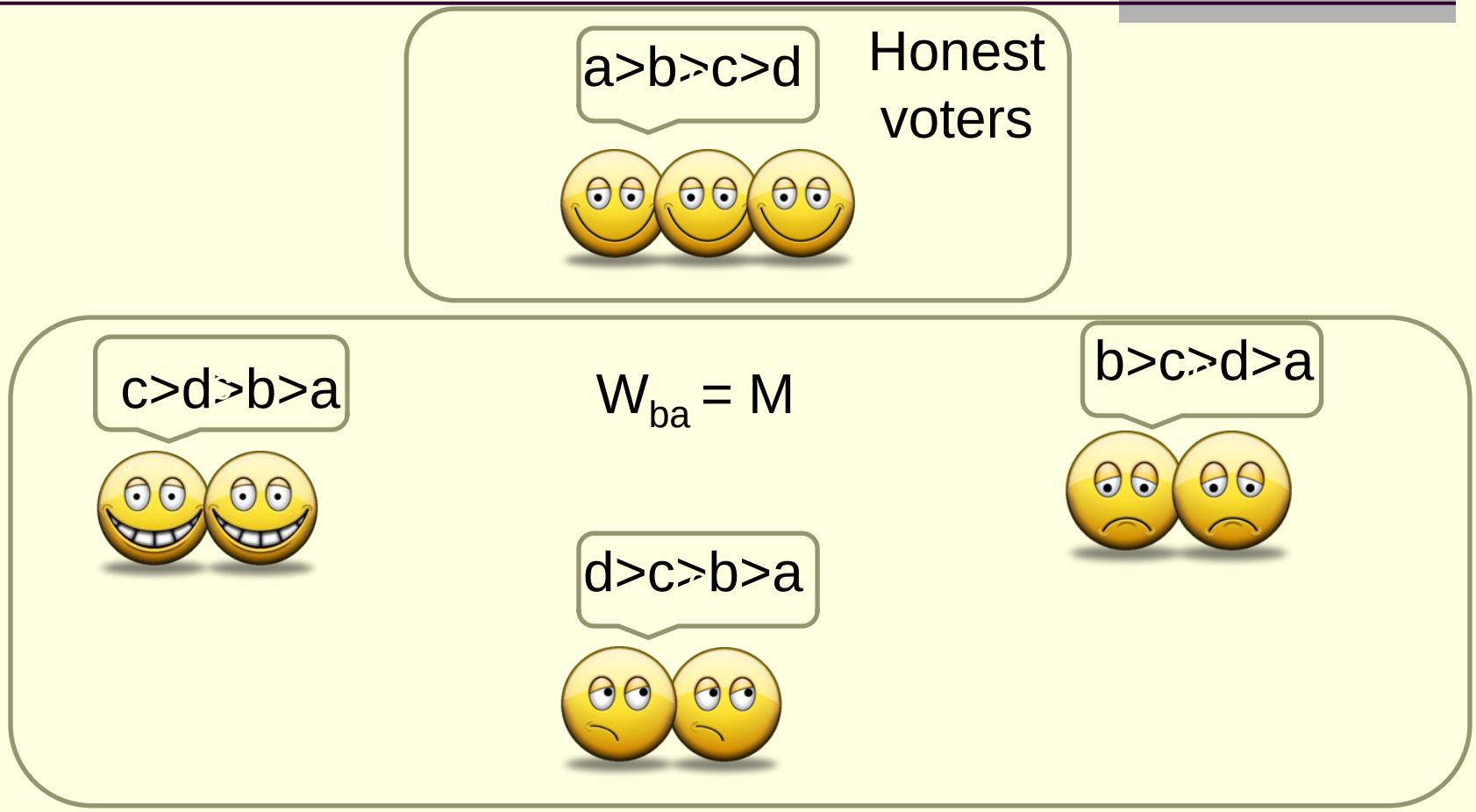
Strong Nash Equilibrium (SNE)

- SNE is a Nash equilibrium in which no coalition, taking the actions of its complements as given, can cooperatively deviate in a way that benefits all of its members
- When a coalition coordinates a deviation, the remaining players are unaware of it → they stick to their agreed-upon strategies

The SNE (more formally)

- Let P_H be a profile of the honest voters
- A profile P'_M is in an SNE if there is no coalition $W \subseteq M$ that can cause an alternative to win that is better for them than $c = R(P_H \cup P'_M)$, when the colluders in $M \setminus W$ vote according to $P'_{M \setminus W}$

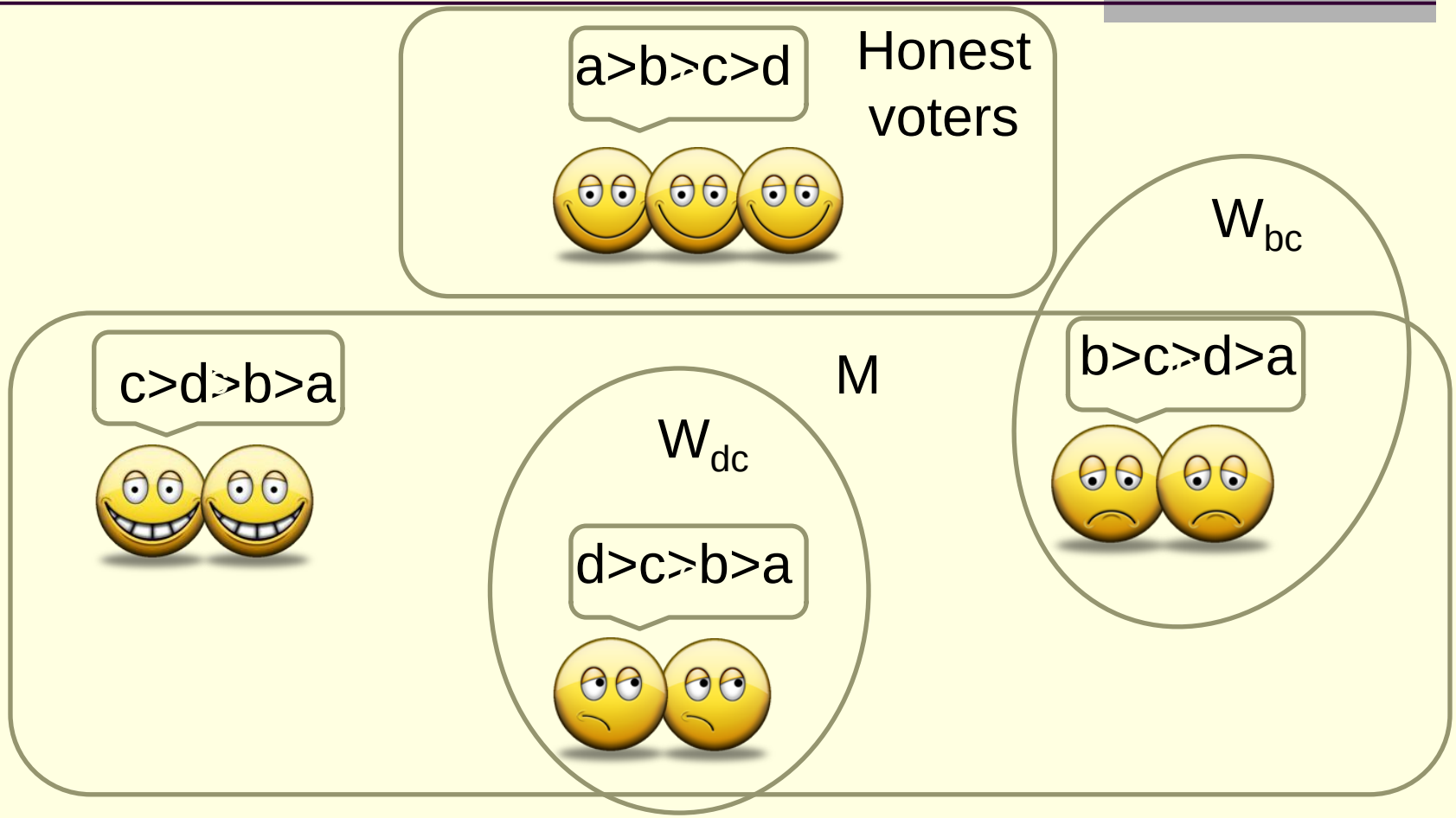
The SNE – example for Plurality - 1/2



Is the profile of truthful preferences an SNE?

No, since everybody in M can deviate and vote for b, for example

The SNE – example for Plurality - 2/2



So, does there exist an SNE?

Yes! When everybody in M votes for c

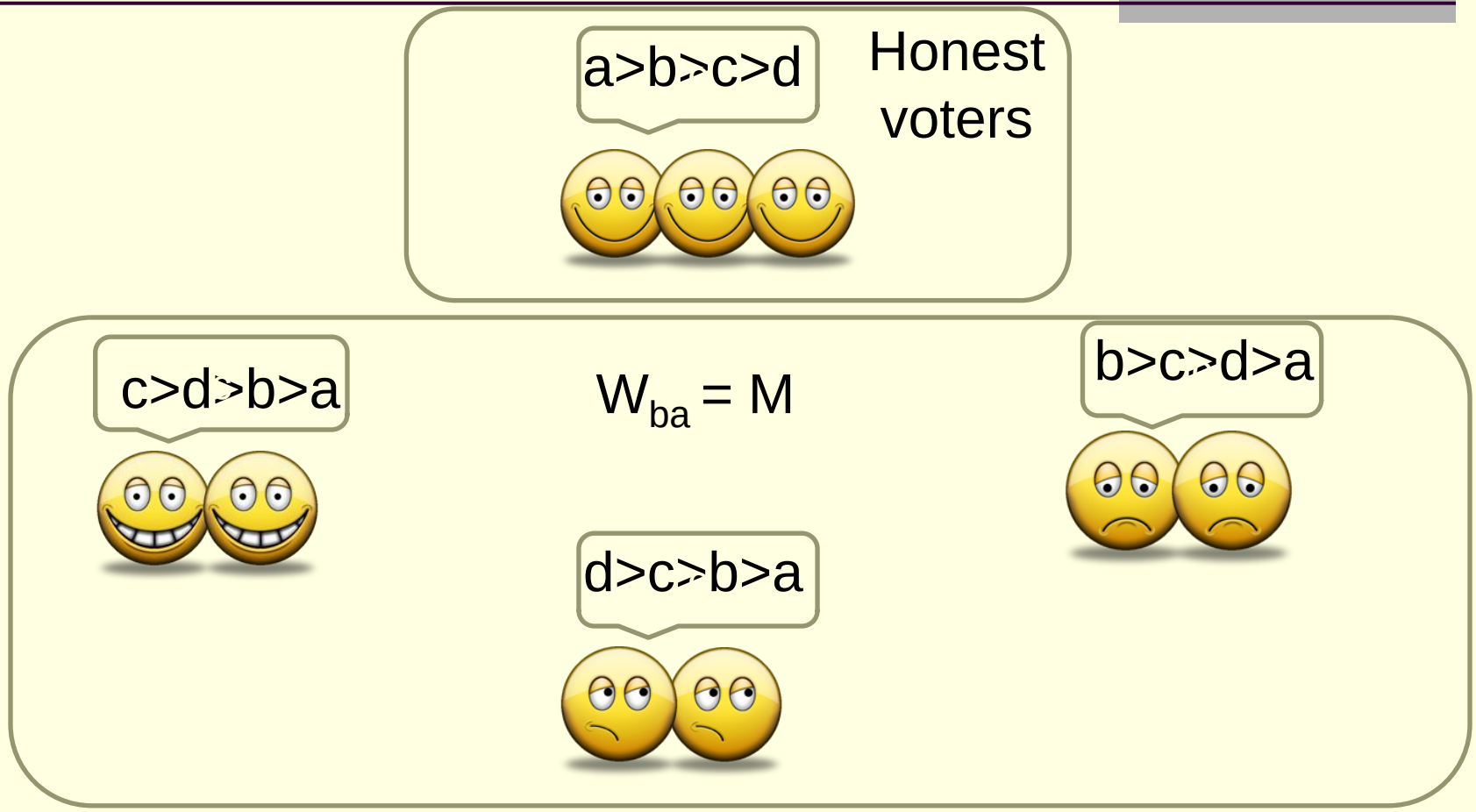
Feasibility

- We say that alternative $c \in C$ is *feasible* for a coalition $W \subseteq M$ if W can make c get elected when everybody in $M \setminus W$ votes truthfully.
- We say that alternative $c \in C$ is *α-feasible* for a coalition $W \subseteq M$ if there exists a profile for W making c get elected for all profiles of $M \setminus W$.
- We say that alternative $c \in C$ is *β-feasible* for a coalition $W \subseteq M$ if for every profile of $M \setminus W$ there exists a profile of W making c get elected.

The Cores

- Let c be an alternative that is feasible for M. We say that c is in a *core* if no coalition $W \subseteq M$ can make a better alternative for them win
  - For α-core we require *α-feasibility*
  - For *β-core* we require *β-feasibility*

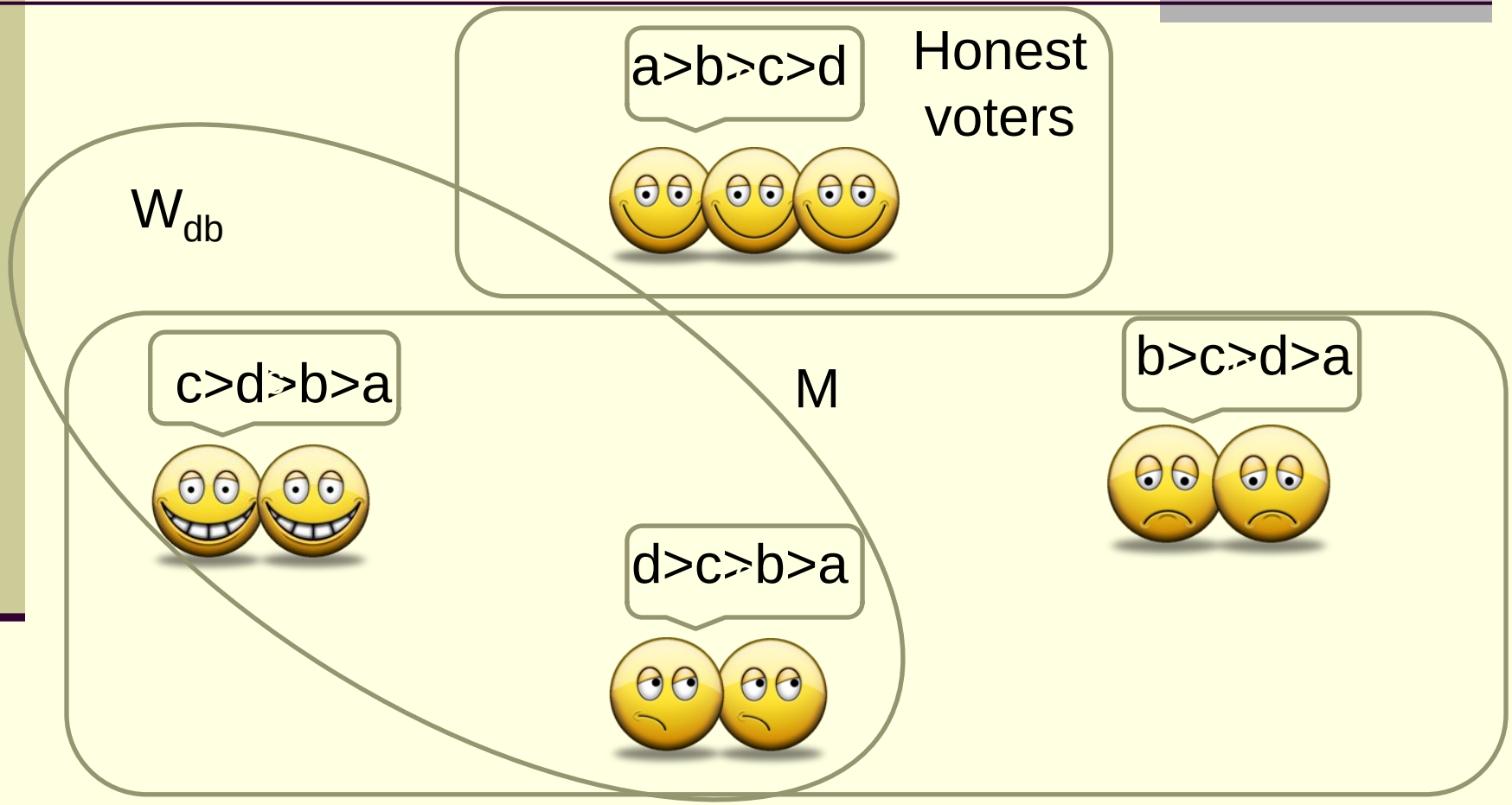
The Cores – example for Plurality - 1/2



Is a∈ α-core?

b is α-feasible for $W_{ba} = M$ → a ∉ α-core

The Cores – example for Plurality - 2/2



Is b∈ β-core?

d is not β-feasible for W_{db}, the same for c, a → b ∈ β -core

SNE vs. β-core

- Lemma: If P'_M is an SNE, then $c = R(P_H, P'_M) \in$ *β-core*
- Proof: If $c \notin$ *β-core* then there exists $c' \neq c$ and $W' \subseteq M$ such that everybody in W' prefers c' to c, and c' is β-feasible for W', then by definition W' can benefit by deviating from $P'_M \rightarrow P'_M$ is not an SNE.

α-core vs. β-core

- Lemma: *β-core* $\subseteq$ α-core
- Proof: Let c be a candidate. If c $\notin$ α-core then there exists a candidate c' ≠ c and a coalition W' $\subseteq$ M such that everybody in W' prefers c' to c, and c' is α-feasible for W' → c' is β-feasible for W' → c $\notin$ *β-core*.

Our results - 1/3

- Theorem 1: The problem of determining whether a given profile P'_M is in an SNE is Turing-reducible to the CCUM problem.
- Proof: Let $c = R(P_H, P'_M)$. For each $x \neq c$, solve the CCUM problem where W_{xc} is the set of manipulators and $P_H \cup P'_{M \setminus W_{xc}}$ are the profiles of the non-manipulators. If for some x and W_{xc} the answer of CCUM is "yes" then P'_M is not an SNE. Otherwise, P'_M is an SNE.

Our results - 2/3

- Theorem 2: The problem of determining whether an alternative c is in the α- or β-core is at least as hard as the CCUM problem.
- Proof: in the reduction from the CCUM, let all the manipulators prefer c. Then c is in the α- and β-core if and only if it is feasible for M.

Our results - 3/3

- Theorem 3: There are polynomial-time algorithms solving the α- and the β-core problems for k-approval and Bucklin voting rules.
- Theorem 4: There is a polynomial-time algorithm solving the Plurality-with-Runoff-β-core problem.

Related work

- The Transferable Utility (TU) setting is studied in “Coalitional Voting Manipulation: A Game-Theoretic Perspective” by Yoram Bachrach, Edith Elkind and Piotr Faliszewski (IJCAI11) [9]
- Peleg and Peters in “Strategic Social Choice” [10] define a rule called “feasible elimination procedure” (f.e.p.)
- F.e.p. elects an alternative which is an outcome of an SNE, and therefore in the α-core and in the β-core

Conclusions

- We explored the stability issue in coalitional manipulation voting games in the NTU setting
- We defined and studied the complexity of the core problems, and an SNE

Future work

- Investigate the complexity of α-core problem in Plurality with Runoff, and α- and β-core problems in Cup voting rule
- What is the complexity, under various voting rules, of finding a profile for the colluders which is an SNE, if one exists?

Summary

- We gave some background on Computational Voting Theory and Algorithmic Game Theory
- We presented algorithms for manipulation problems under Maximin
- We briefly discussed
  - Transformation Games
  - Effort Games
  - Manipulating Quota in WVGs
- We explored manipulation from cooperative game theoretic point of view

Thank you!

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Backup slides

Manipulation with Randomized Tie-Breaking under Maximin

AAMAS'12

Joint work with Jeff Rosenschein

The classic manipulation problem

- Given
  - A voting rule *R*
  - The Profile of Non-Manipulators *PNM*
  - Candidate *c* preferred by the (single) manipulator
- We are asked whether or not there exists a preference order *L* for the manipulator such that *c* is the winner of *PNM* ∪ *{L}* under *R.*

Our setting

- $C = \{c_1,\ldots,c_m\}$ – the set of candidates
- $V = \{1,\ldots,n\}$ – the set of voters
- $V' = \{1,\ldots,n-1\}$ – the set of non-manipulators
- The voter n is the manipulator
- The Maximin score of c is:

  $$\min_{c' \neq c} |\{i \mid c >_i c',\ i \in V\}|$$

- The candidates with maximum score win

Notation

- $N(c, c') = |\{ i \mid c >_i c', i \in V'\}|$ – the number of non-manipulators who prefer c to c'
- $S(c) = \min_{c' \neq c} N(c,c')$ – the Maximin score of c from the non-manipulators

Tie-breaking

- Most of the literature assumes tie-breaking relative to the manipulator
  - In favor of manipulator, or
  - Adversarial to the manipulator
- Sometimes some lexicographic tie-breaking is assumed

Randomized tie-breaking

- We assume that ties are broken uniformly at random
- Suppose that manipulator's preference is

  $c_1 > c_2 > \ldots > c_m$
- What is better for the manipulator: c_1 tied with c_m, or c_2?
- Need a notion of utilities

Utility of the manipulator

- $u(c) \in \{0,1\}$ – the manipulator's utility for candidate c
- We assume that the utilities are compatible with the manipulator's preferences
- Let S be the set of the winning candidates
- Then the manipulator's expected utility is

$$\hat{u}(S) = \frac{1}{|S|}\sum_{c \in S} u(c)$$

Manipulation problem with randomized tie-breaking (MRTB)

- Given
  - A voting rule *R* (here: Maximin)
  - The Profile of Non-Manipulators *PNM*
  - The manipulator's utilities of the candidates $u(c_j)$
  - A rational number $q \geq 0$
- We are asked whether or not there exists a vote L for the manipulator such that

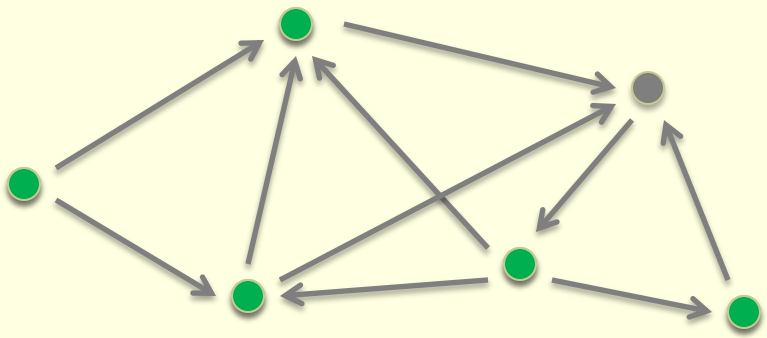
  $$\hat{u}(R(PNM \cup \{L\})) \geq q$$
- In the optimization version we are asked to find the optimal vote for the manipulator

Related work

- Obraztsova et al. (AAMAS'11) showed an algorithm for finding an optimal manipulation under Maximin when the utilities are (1, 0,…, 0)
- Obraztsova and Elkind (IJCAI'11) proved that this problem is NP-hard when the utilities are (1, …, 1, 0)
- What if the utilities are (1, …, 1, 0, …, 0) with k 1's?

Maximum Directed Acyclic Subset (Max-DAS) Problem

- Given digraph G = (W, E) we are asked to find a maximum subset of vertices $S \subseteq W$ which induce acyclic sub-graph of G



Parameterized Complexity

- Classifies problems' complexity as a function of multiple parameters of the input
- Finer scale classification of NP-hard problems
- A parameterized problem is a language $L \subseteq \sum^* \times \mathrm{N}$
- *L* is called fixed-parameter tractable (in FPT) if the question "$(x, k) \in L$?" can be decided in time $f(k) \cdot |x|^{O(1)}$

Main result

- Suppose the utility vector of the manipulator is (1,…,1,0,…,0) with k 1's
- Theorem 1: If there is α–approx. algorithm for Max-DAS problem on graph with k vertices running O(f(k)) time, then there is α–approx. algorithm for MRTB under Maximin running $O(f(k) + (n+m)m^2)$ time
- Corollary 1: MRTB under Maximin can be done in time $O(2^k k^2 + (n+m)m^2)$ (➔ in FPT)
- Corollary 2: When k = O(log(m) + log(n)) then it is in P

Election Graph

- We define the election digraph $G = (W, E)$ as follows:
  - $W = C$
  - $(c, c') \in E$ iff $N(c', c) = S(c')$
- Let $X = \text{argmax}_c\{S(c) \mid u(c) = 1\}$
- Let $H = (X, E')$ be the sub-graph of G induced by vertices X

Proof sketch of the Theorem 1

- First run the algorithm of Obraztsova et al. for utilities vector (1,0,…,0) to minimize the number of winning candidates with utility 0
  - Get $L_1 :: L_2$, where L_1 is some order on X
- Then run Max-DAS approximation algorithm on the digraph H = (X, E'); let $S \subseteq X$ be its output
- Topologically sort the vertices of S: $[x_1,\ldots,x_{|S|}]$
- Return $[x_{|S|},\ldots,x_1] :: L' :: L_2$, where L' is some order on X \ S

An approximation result

- Lemma: If in digraph G = (W, E) there are at least |W|/b vertices having in-degree or out-degree ≤ b’, then we can approximate Max-DAS problem up to the factor of b(b’+1).
- Corollary: If in the digraph H at least |X|/b vertices have in-degree or out-degree ≤ b’, then we can approximate the optimal expected utility up to the factor of b(b’+1).

Approximation algorithm for Max-DAS problem

- $S = \emptyset$
- While $W \neq \emptyset$
  - Let $x = \text{argmin}_{v \in W}\{\min\{d_{in}(v), d_{out}(v)\}\}$
  - $S = S \cup \{x\}$
  - If $d_{in}(x) \leq d_{out}(x)$ then remove x from W together with all the incoming neighbors of x
  - Else remove x from W together with all the outgoing neighbors of x
- Return S

Proof of Lemma – 1/2

- The set S returned by the algorithm induces acyclic sub-graph G’ = (S, E’):
  - Suppose for contradiction there is a cycle T in G’
  - Arrange the vertices of S on a line from left to right in the order of addition to S
  - Let a be the leftmost vertex of T
  - a has an edge to the right and an edge from the right, a contradiction

Proof of Lemma - 2/2

- If at least $|W|/b$ of the vertices have in-degree or out-degree $\leq b'$, then the above algorithm gives $b(b'+1)$-approximation to the Max-DAS problem:
  - When processing the first $|W|/b$ vertices, with every vertex added to S we delete at most $b'+1$ vertices from W.
  - Therefore, we will do at least $|W|/(b(b'+1))$ iterations of the main loop
  - I.e., we will add at least $|W|/(b(b'+1)) \geq OPT/(b(b'+1))$ vertices to S

An exact result

- Theorem 2: If in the digraph H = (X, E') induced by the set $X = \text{argmax}_c\{S(c) \mid u(c) = 1\}$ there is a 2-cycle, then the approximation algorithm attains the optimum
- Proof idea: We prove that when there is a 2-cycle in H, then H is complete
- Therefore, in any permutation of X, there is only one candidate in X whose score increases

Conclusions

- We defined the manipulation with randomized tie-breaking problem
- We proved that when the utilities of the manipulator are (1,…,1,0,…,0) with k 1's, it is in FPT when parameterized by k
- We identified conditions which allow to approximate MRTB up to a constant factor
- We investigated when this problem could be solved exactly and efficiently
- What about general utilities?

Proof Systems and Transformation Games

CoopMAS'10

Joint work with Yoram Bachrach, Michael Wooldridge and Jeff Rosenschein

Transformation Games - Introduction

- Cooperative game
- Players are endowed with
  - initial resources
  - capabilities of deriving certain output resources given certain input resources
- The aim is to generate a particular target resource
- It is achieved by forming coalitions

Example 1 – supply chains

- Firm A – drills for crude oil
- Firm B – produces refined oil from crude oil
- Firm C – packages refined oil in barrels
- Firm D – transforms refined oil into petrol sold to consumers

Questions for supply chains

- Which supply chains are more likely to be formed?
- How should the firms share the revenue?
- Which firm is most important in the supply chain?

Example 2 – cooperative proof systems

- A set of experts tries to prove a complex theorem
- Each expert knows facts and proofs for simpler theorems
- Suppose they can prove the theorem together
- How can they quantify the contribution of each expert?

Notation

- $I = \{a_1,\ldots,a_n\}$ – is the set of agents
- $R = \{r_1,\ldots,r_k\}$ – is the set of resources
- $r_g \in R$ is the goal resource
- $R_i \subseteq R$ is the resource set of a_i
- A transformation is a pair <B, r> where $B \subseteq R$, $r \in R$
- D is the set of all possible transformations over R
- $D_i \subseteq D$ are the transformations of ai

Assumptions

- We do not model resource *quantity*
- We do not model resource *consumption*
- In the basic model there are no *costs* associated with transformations
  - The model extension contains such costs

Transformations

- $\Rightarrow \subseteq 2^I \times R$ – is a relation where $C \Rightarrow r$ means that coalition C can produce resource r
  - Defined recursively
- TG Γ with a goal resource r_g induces a simple TU coalitional game:

$$v_\Gamma(C) = \begin{cases} 1 & \text{if } C \Rightarrow r_g \\ 0 & \text{otherwise} \end{cases}$$

- It is called *Unrestricted* TG (UTG)

Restrictions on TGs

- Definition: A *Transformation restricted* TG (DTG) with goal resource r_g and transformation bound k is the game where coalition C wins if it can derive r_g using at most k transformations, and loses otherwise
- Definition: A *Time limited* TG (TTG) with goal resource r_g and time limit t is the game where coalition C wins if it can derive r_g with time of at most t, and loses otherwise
  - Allows for simultaneous transformations

Results

Problem	UTG	DTG	TTG
CV	P	P (NPH)	P
VETO	P	P (co-NPH)	P
DUMMY	co-NPC	co-NPC(co-NPH)	co-NPC
CORE	P	P (co-NPH)	P
SHAPLEY	co-NPH	co-NPH	co-NPH
BANZHAF	#P-Hard	#P-Hard	#P-Hard

- If the results differ for simple and complex transformations, the results for complex transformations are given in parentheses

TGs with associated costs

- Every transformation d has cost $c(d) \in \Re^+$
- h(C, r) is the minimum cost needed to obtain r from R_C.
  - The sum of trans. costs in the minimum sequence of trans. from R_C to r.
- $v(r_g) \in \Re^+$ is the value of the goal resource
- Definition: TGs *with costs* (CTG) with the goal resource r_g and cost function $c: D \rightarrow \Re^+$ is the game where:

$$v(C) = \max(0, v(r_g) - h(C, r_g))$$

Results for CTGs

Problem	CTGs
Coalitional Value (CV)	P
DUMMY	co-NPC
SHAPLEY	co-NPH
BANZHAF	#P-Hard

Conclusions

- We introduced Transformation Games
- TGs are kind of cooperative games
- Can model cooperative proof systems and supply chains
- We defined some restrictions on TGs
- Studied computational complexity of various related problems
- Extended the model by adding costs to transformations

Future Work

- Extend the model to consumable resources
- More refined solution concepts:
  - Least core
  - Nucleolus
- Uncertain domains: domains where transformations may fail