The Big Equilibria Cull

Advances in the effort to make Nash equilibria relevant in voting games

Joint work with David R.M. Thompson, Kevin Leyton-Brown, Reshef Meir, Zinovi Rabinovich, Svetlana Obraztsova, Evangelos Markakis and Jeffrey S. Rosenschein.

What is voting?

Deciding which is the best muppets character…

Waldorf	Statler	Jim
		
		
		

What is voting? plurality

There is no single correct way to vote: Many methods could be devised.

In our example, if each voter just voted for his favorite (plurality), we would need a **tie-breaking rule**.

(e.g.:
K vs. *P*⇒*P*;
K vs. *S*⇒*S*;
P vs. *S*⇒*S*;
K vs. *P* vs. *S*⇒*K*)

What is voting? scoring rules

Scoring rules for m candidates define a scoring vector:

$$(\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_m)$$

under the condition

$$\alpha_1 \geq \alpha_2 \geq \alpha_3 \geq \ldots \geq \alpha_m = 0$$

A voter gives α_1 points to his most preferred candidate, α_2 points to his 2nd preference, etc.

The winner is the candidate with most points

What is voting? scoring rules

Plurality: $(1,0,\ldots,0,0)$

Veto: $(1,1,\ldots,1,0)$

Borda: $(m\text{-}1,m\text{-}2,\ldots,1,0)$

k candidates

***k*-approval:** $(1,1,\ldots,1,0,0,\ldots,0)$

k candidates

***k*-veto:** $(1,1,\ldots,1,0,0,\ldots,0)$

What is voting? Condorcet

But consider every 2 candidates had a "one on one" contest:

Statler

Jim

Miss Piggy

vs.

Waldorf

Kermit

Waldorf

Jim

Miss Piggy

vs.

Statler

Swedish chef

Miss Piggy wins against any other option!

What is voting? Gibbard-Satterthwaite

Voting has dark secret…

What is voting? Gibbard-Satterthwaite

A non-dictatorial voting method, in which there is, for every candidate, some set of votes which enable it to win, must be susceptible to manipulation, i.e., a voter (with full knowledge of others' votes) will find it beneficial not to vote according to his true preferences

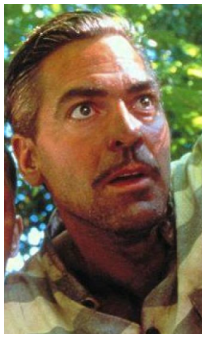
What is a Nash Equilibrium?

So we turn to a different option…

A solution concept involving games where *all players know the strategies of all others*. If there is a set of strategies with the property that no player can benefit by changing her strategy while the other players keep their strategies unchanged, then that set of strategies and the corresponding payoffs constitute the **Nash Equilibrium**.

What is a Nash Equilibrium?

Example: voting prisoners' dilemma…

	 **Everett**	 **Pete**	 **Delmar**
1st preference	Escape 	Riot 	Stay in prison 
2nd preference	Riot 	Escape 	Escape 
3rd preference	Stay in prison 	Stay in prison 	Riot 

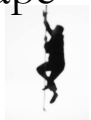
What is a Nash Equilibrium?

Example: voting prisoners' dilemma…

Everett

Pete

Delmar

	Everett	Pete	Delmar
1st preference	Escape 	Riot 	✗ Stay in prison 
2nd preference	Riot 	Escape 	Escape 
3rd preference	✗ Stay in prison 	✗ Stay in prison 	Riot 

The truth shall set you free

What is truth-bias?

Adding a **Truthfulness incentive**, which adds a small ε to the utility of each player when it votes truthfully. The ε is small enough such that voters will still manipulate if they can.

What is truth-bias? Example

Everett

Pete

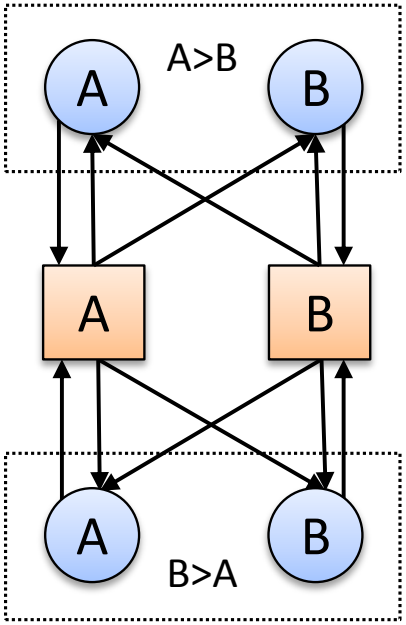
Delmar

	Everett	Pete	Delmar
1st preference	Escape	Riot	Stay in prison
2nd preference	Riot	Escape	Escape
3rd preference	Stay in prison	Stay in prison	Riot

Action Graph Games

A compact way to represent games with 2 properties:

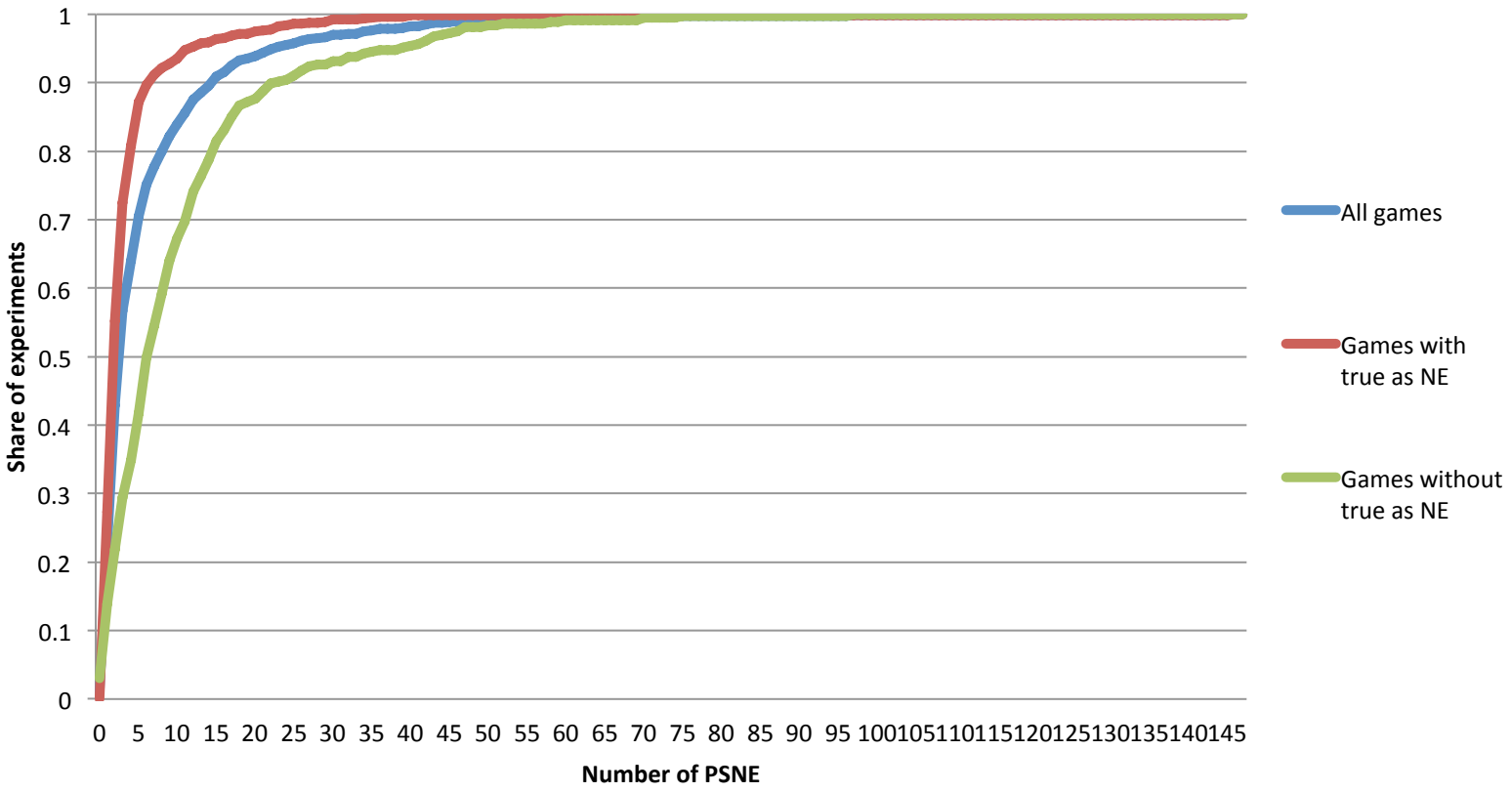
Anonymity: payoff depends on own action and *number* of players for each action.



Context specific independence: payoff depends on easily calculable statistic summing other actions.

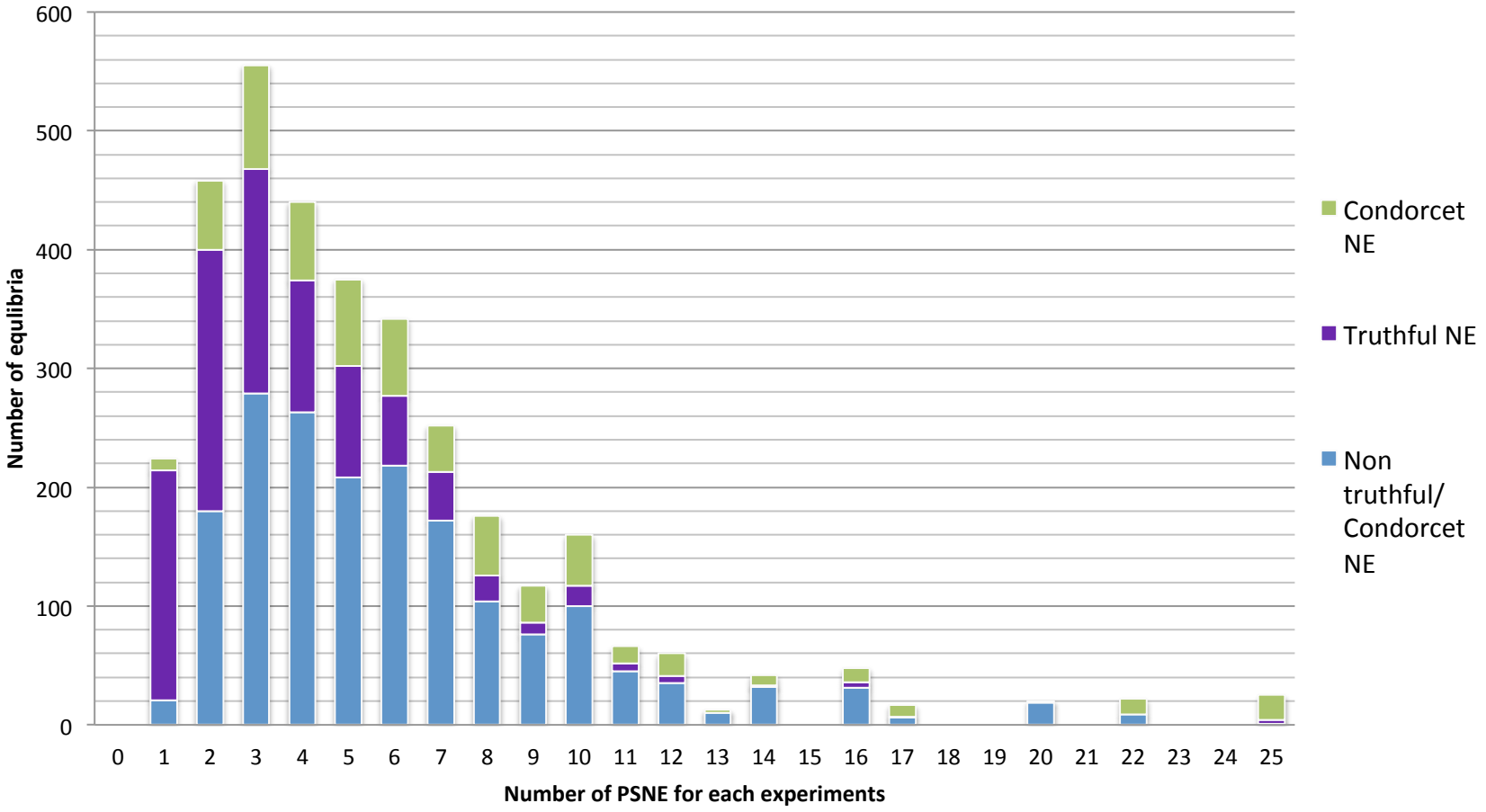
Calculating the equilibria using *Support Enumeration Method* (worst case exponential, but thanks to heuristics, not common).

Truth bias: number of equilibria



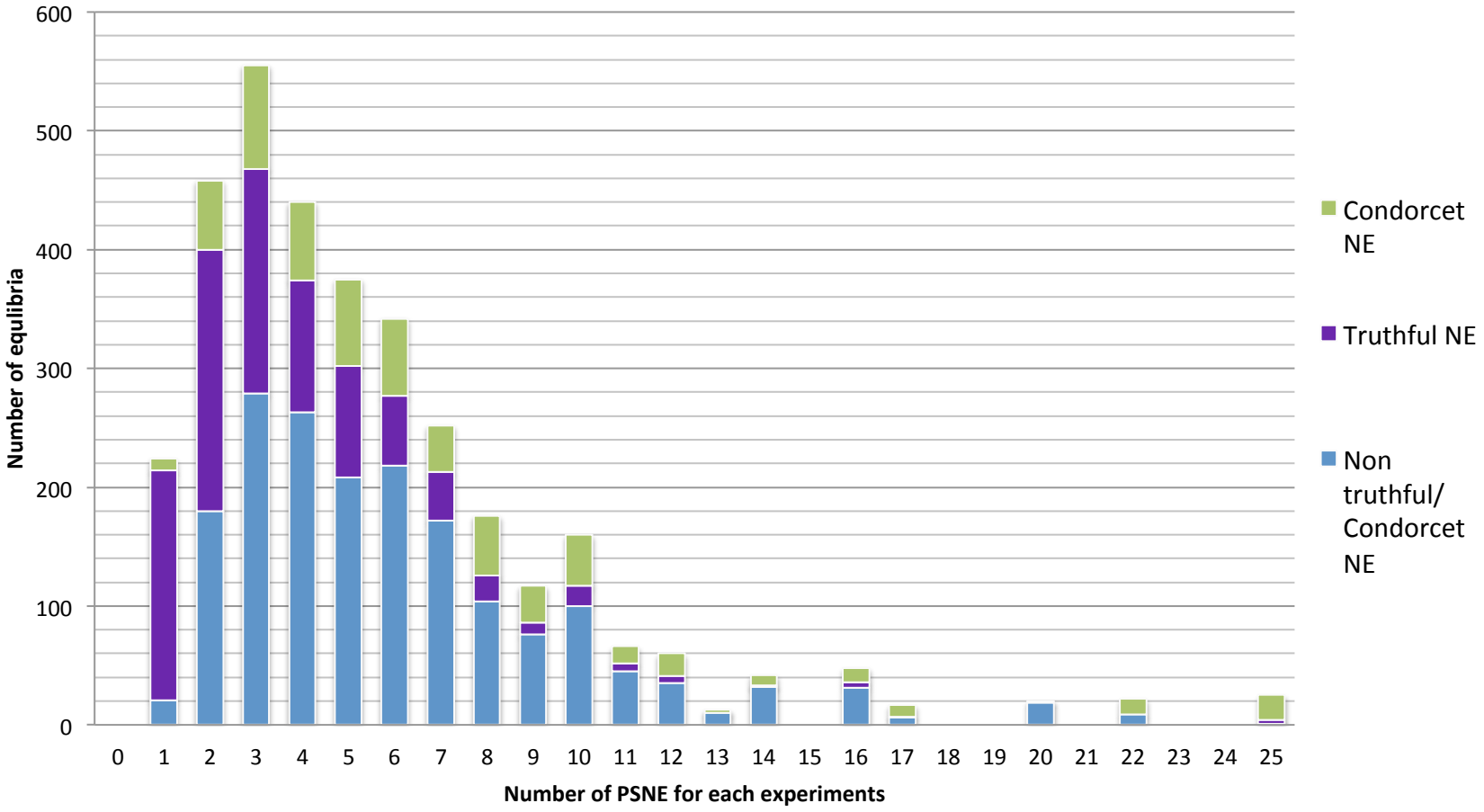
In 63.3% of games, voting truthfully was a Nash equilibrium.
96.2% have less than 10 pure equilibria (without permutations).
1.1% of games have **no pure Nash equilibrium** at all.

Truth bias: type of equilibria – truthful



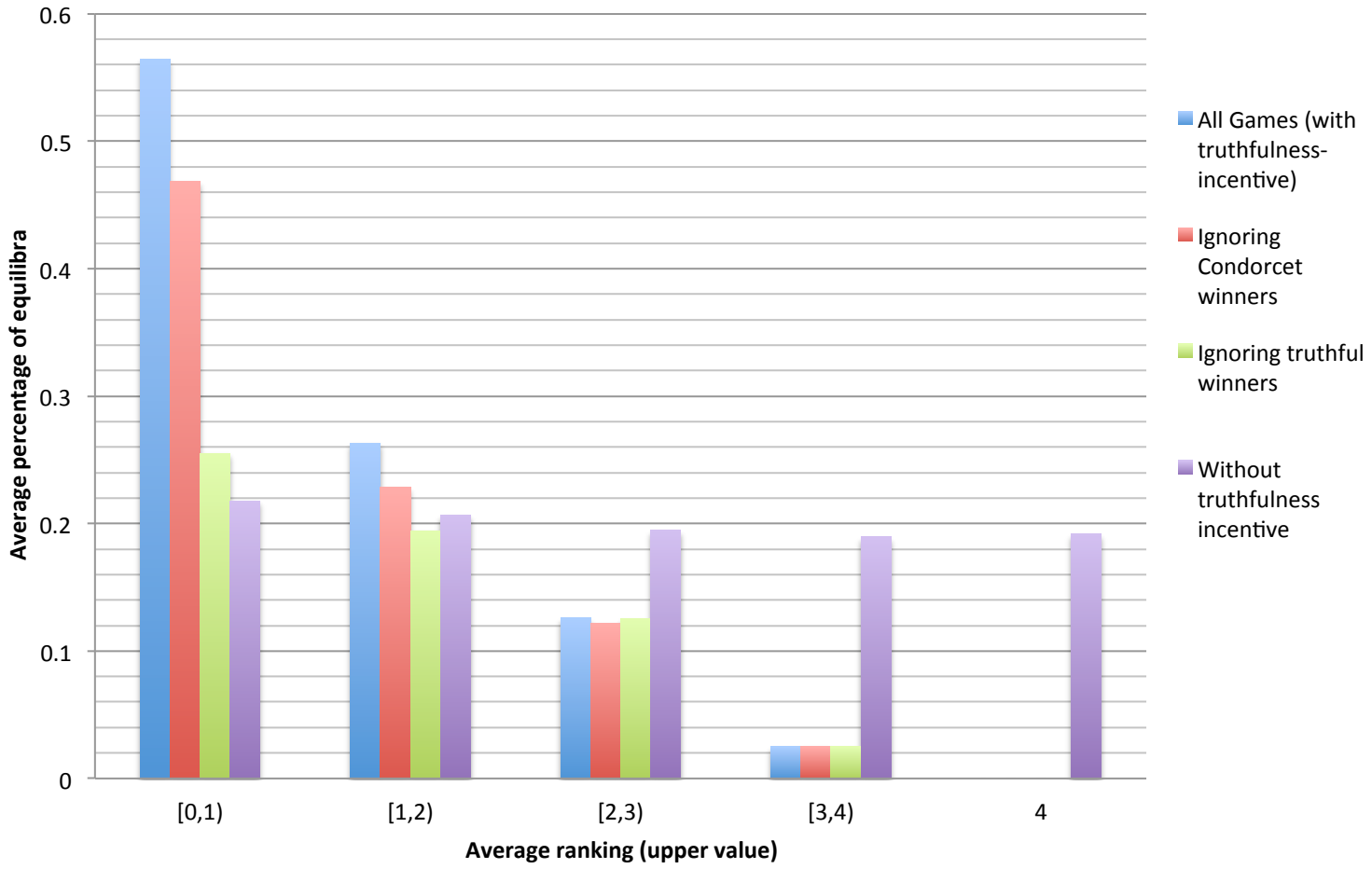
80.4% of games had at least one truthful equilibrium. Average share of truthful-outcome eq.: 41.56% (without incentive: 21.8%).

Truth bias: type of equilibria – condorcet



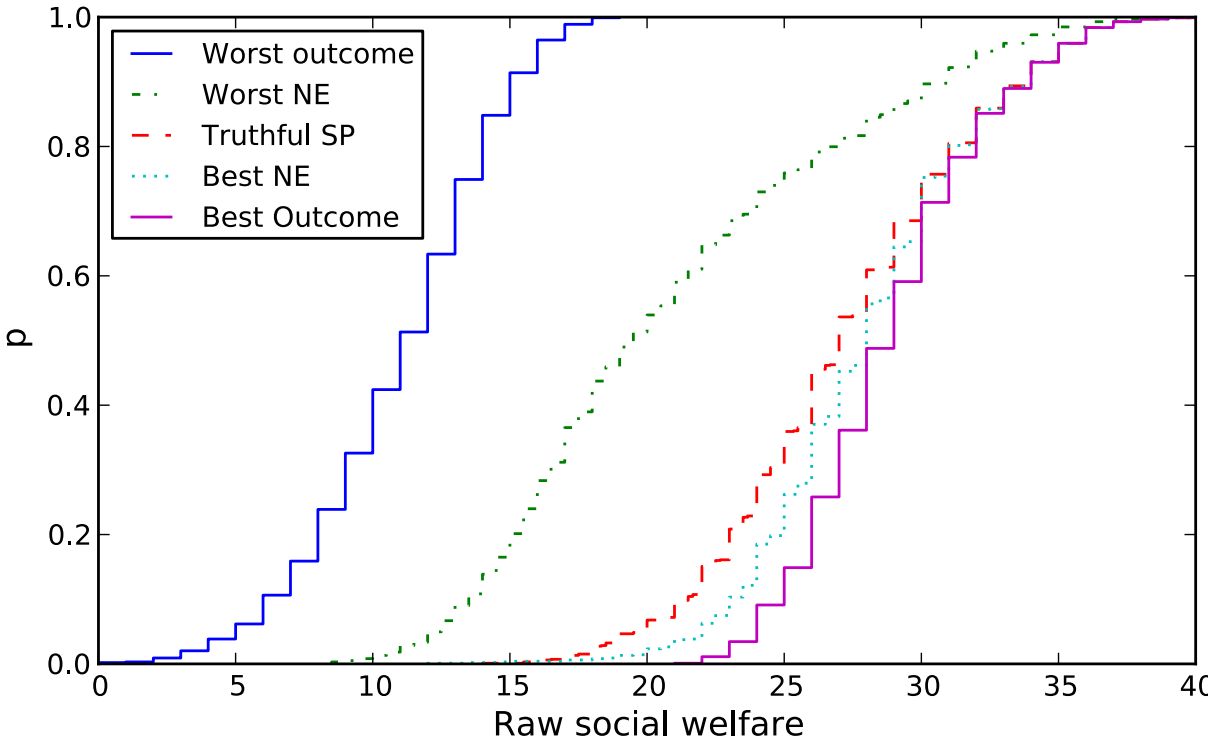
92.3% of games had at least one Condorcet equilibrium.
Average share of Condorcet equilibrium: 40.14%.

Truth bias: social welfare – average rank



71.65% of winners were, on average, above median.
52.3% of games had *all* equilibria above median.

Truth bias: social welfare – raw sum



92.8% of games, there was no pure equilibrium with the worst result (only in 29.7% was best result not an equilibrium).
59% of games had truthful voting as best result (obviously dominated by best equilibrium).

Truth biased plurality

All non-truthful voters vote for the winner.

It is NP-complete to know if there is a Nash equilibrium with truth-biased voters using plurality.

Obratzova et al., SAGT 2013

Truth biased veto

The winner will have the same score (and vetoing voters) as in a truthful vote. Non-truthful ones vote against a runner-up

It is NP-complete to know if there is a Nash equilibrium with truth-biased voters using veto.

Truth biased *k*-approval?

No one necessarily keeps their score…

Laziness shall set you free?

What is lazy-bias?

Adding a **laziness incentive**, which adds a small ε to the utility of each player when it abstains. The ε is small enough such that voters will still manipulate if they can.

Desmedt & Elkind, EC 2010

Going around in circles

What is Iterative Voting?

Color of the new car…

Kermit:

Piggy:

Fozzie:

Gonzo:

Scooter:

(Scooter breaks ties)

What is Iterative Voting?

Color of the new car…

Kermit:

Piggy:

Fozzie:

Gonzo:

Scooter:

What is Iterative Voting?

Color of the new car…

Kermit:

Piggy:

Fozzie:

Gonzo:

Scooter:

What is Iterative Voting?

Color of the new car…

Kermit:

Piggy:

Fozzie:

Gonzo:

Scooter:

Can’t we all just get along?

Beginning the analysis…

Assuming game uses *plurality* with a *linear tie-breaking* rule and players are *myopic* and pursue *best response* strategy, **iterative plurality** converges to a Nash equilibrium.

(best response is critical)

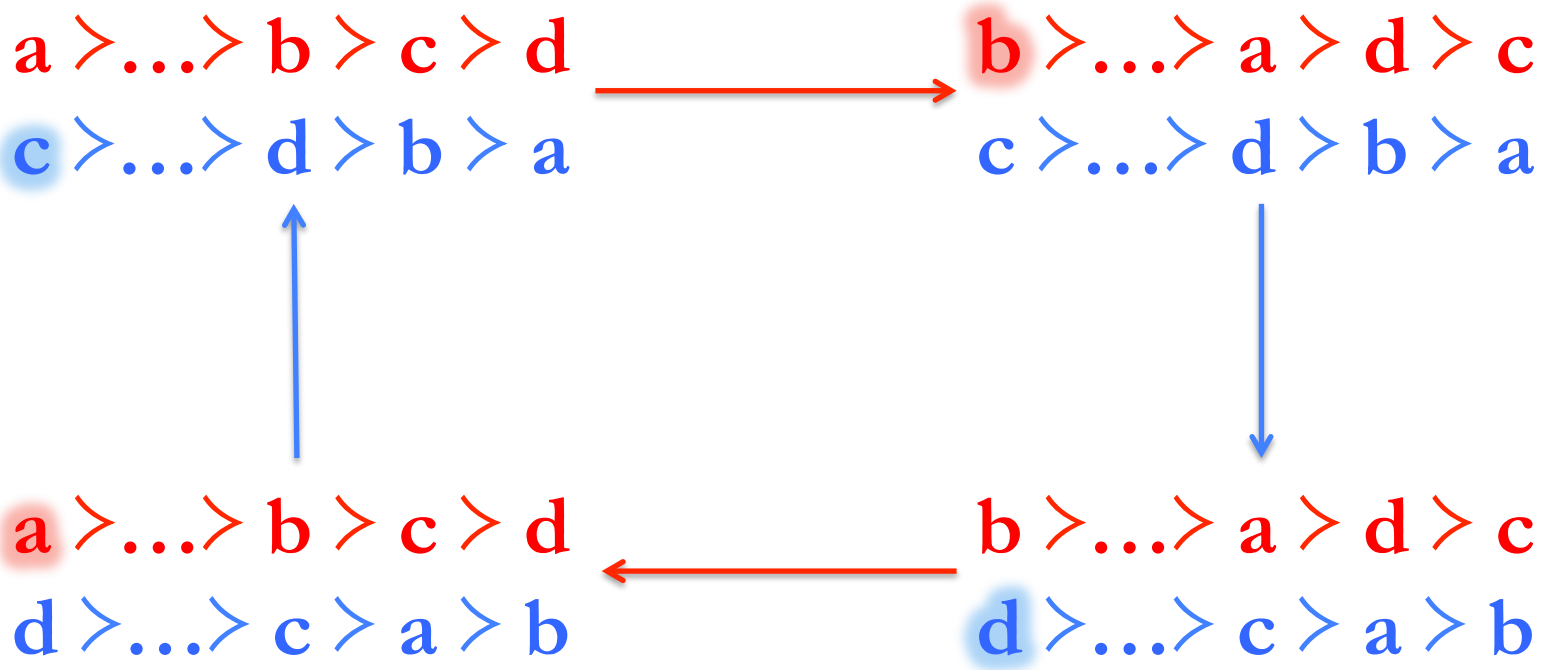
Meir et al., AAAI 2010

When does convergence happen? Tie-breaking rules

If we allow all tie-breaking rules, no scoring rule will converge

When does convergence happen? Tie-breaking rules partial proof

4 candidates, 2 voters, tie breaking rule makes *c* win if not tied with *b*. *b* wins if not tied with *d*. *d* wins if not tied with *a*.

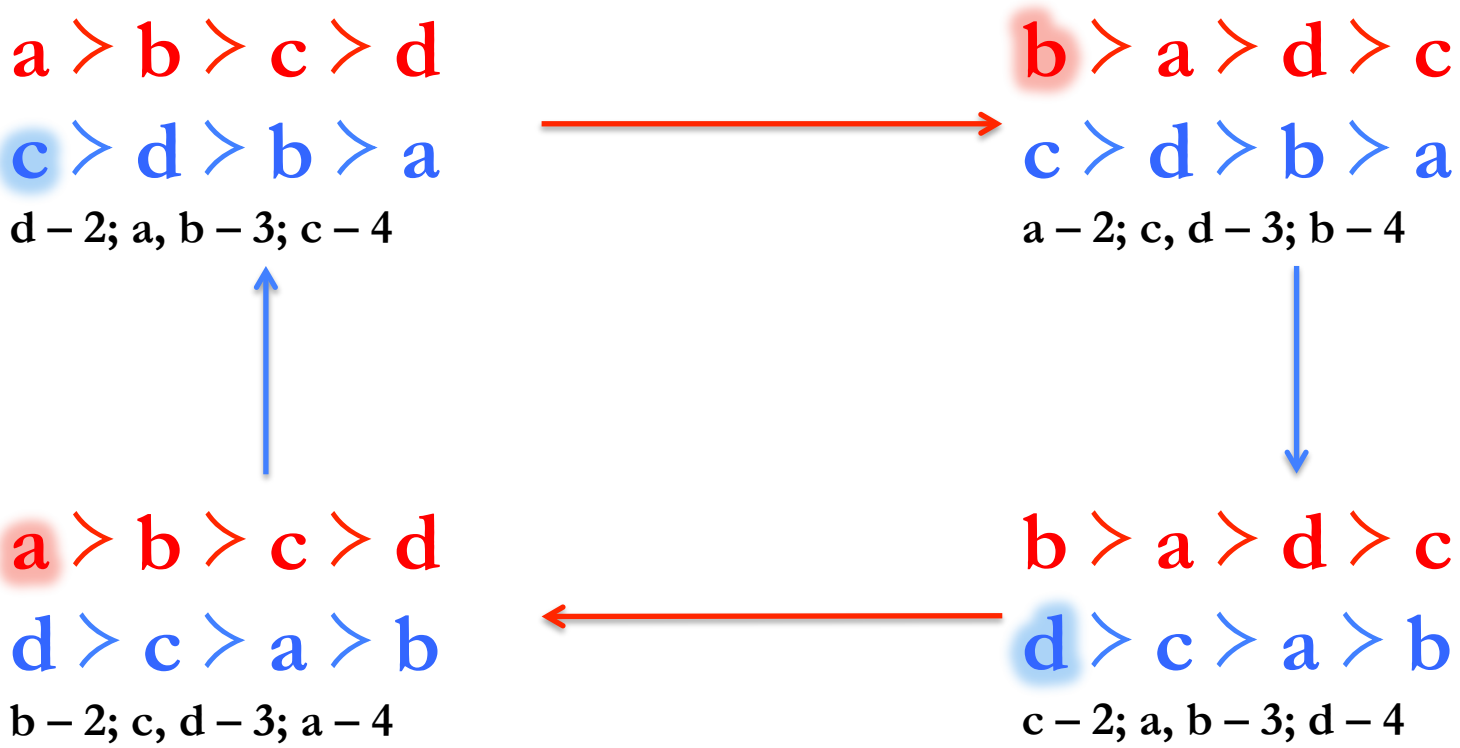


When does convergence happen? Voting rules

Among scoring rules, only plurality and veto converge.

When does convergence happen? Voting rules (Borda example)

4 candidates, 2 voters (tie breaking doesn't matter):



What about the equilibria?

Finding if an equilibria is reachable from the truthful state is NP-complete.

Mix it all together

Does it converge? Iterative truth-bias

Color of the new car…

Kermit:

Piggy:

Fozzie:

Gonzo:

Scooter:

Does it converge? Iterative truth-bias

Color of the new car…

Kermit:

Piggy:

Fozzie:

Gonzo:

Scooter:

How do equilibria look like?

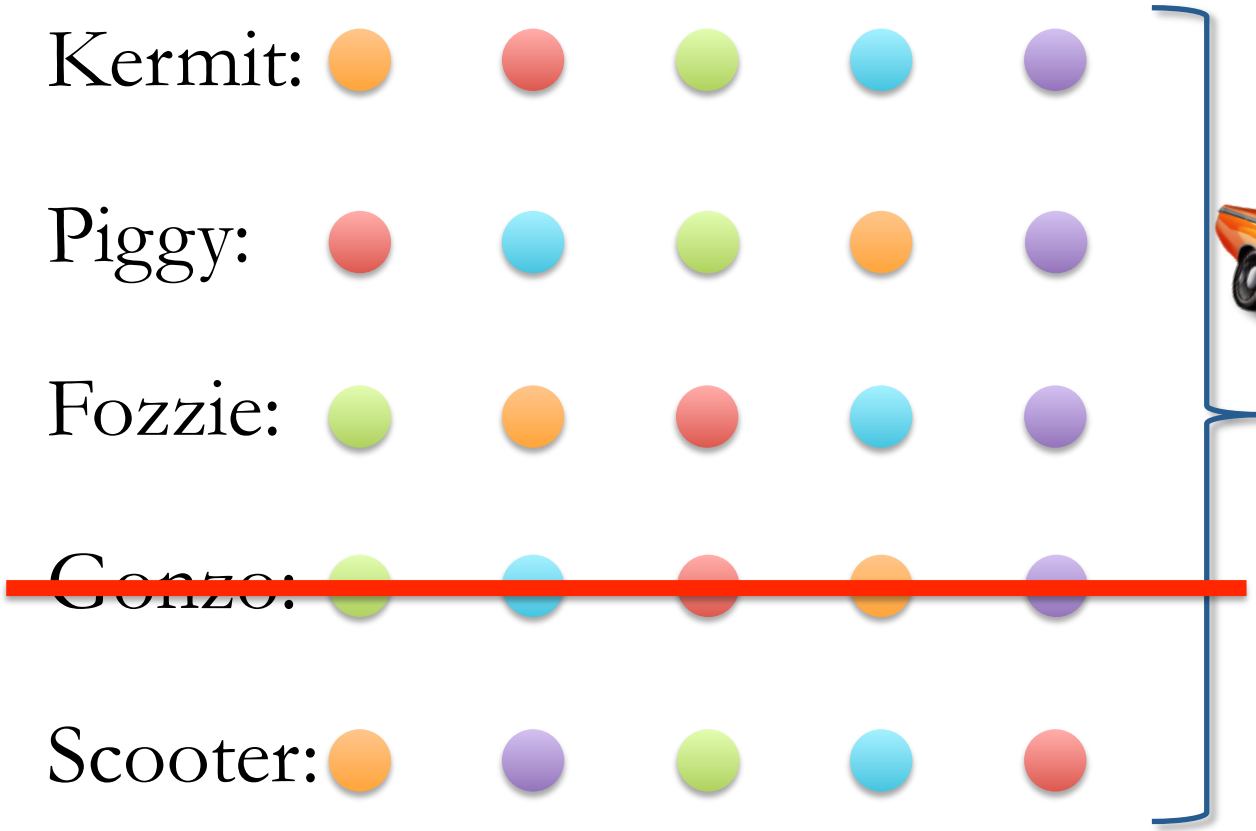
Iterative truth-bias

Only one voter, at most, is untruthful

There is a O(mn) algorithm to find all Nash equilibria starting from truthful state

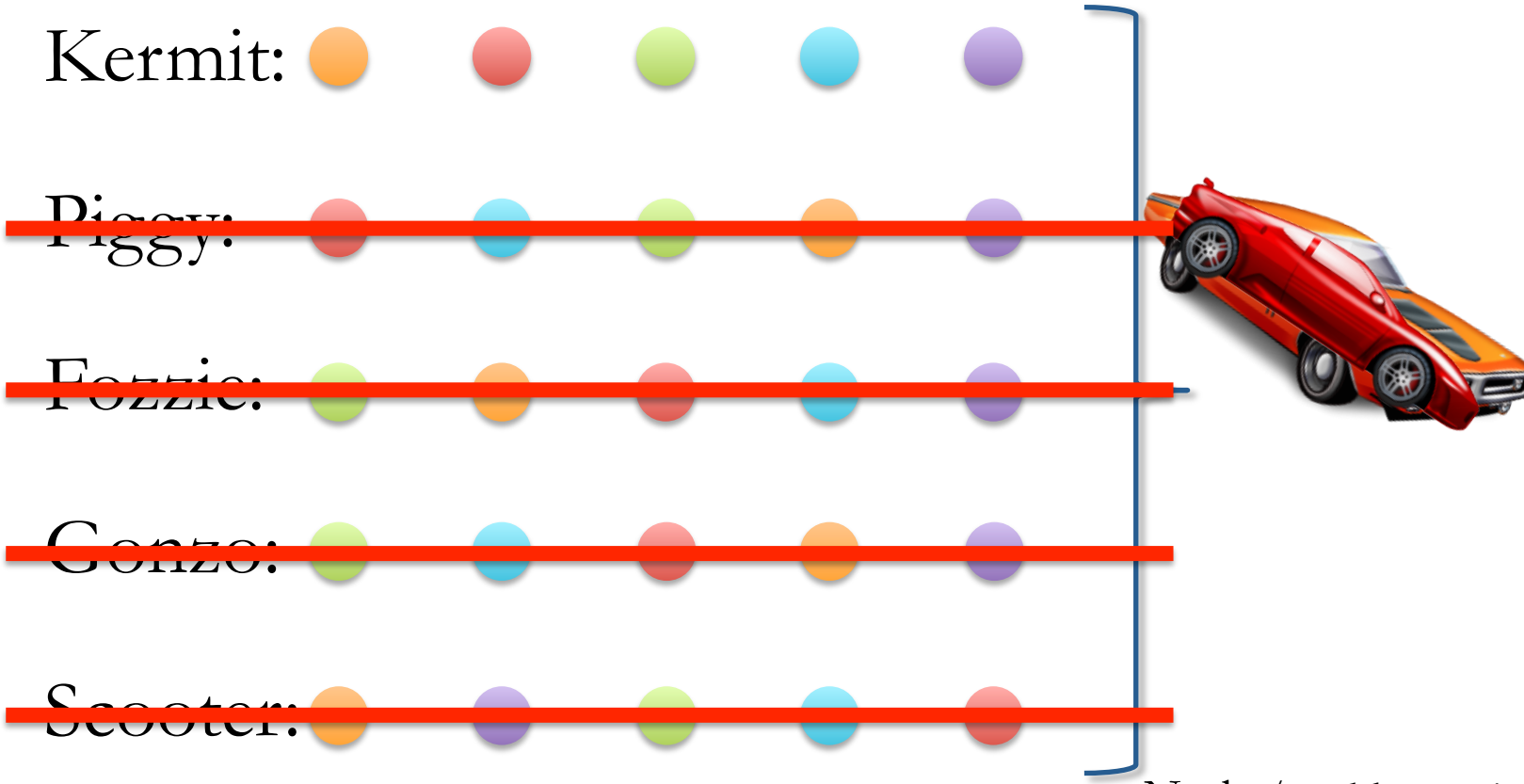
Does it converge? Iterative lazy-bias

Color of the new car…



Does it converge? Iterative lazy-bias

Color of the new car…



Nash ≠ stable state!

How do equilibria look like?

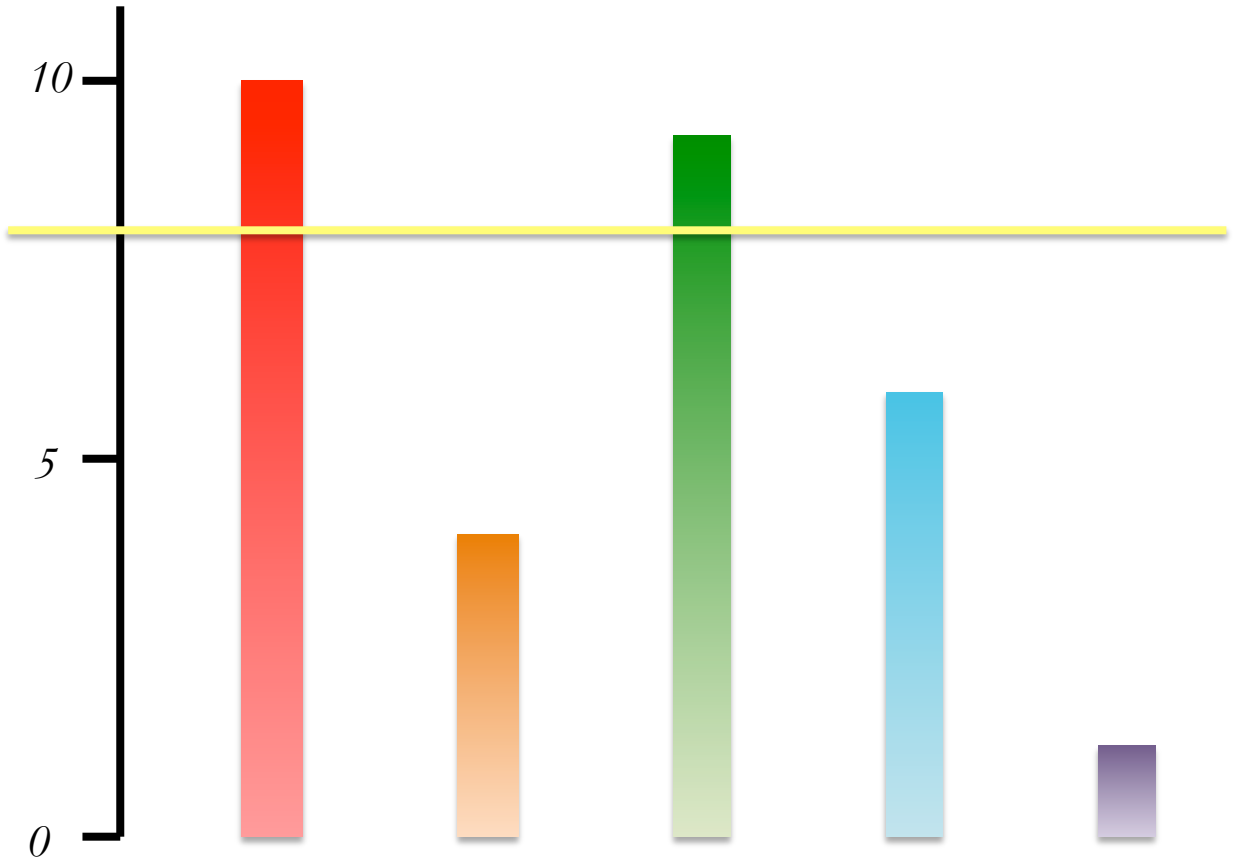
Iterative lazy-bias

All voters abstain but one

There is a polynomial algorithm to find all Nash equilibria starting from truthful state

Now, let's add some mystery…

Radius of uncertainty



Local dominance general games

Define a **metric** on the states.
When in state **s**, consider all states within a radius *r* of **s** to be possible.
Strategy *b* locally dominates strategy *a* if for every strategy **s'** such that $|\mathbf{s}-\mathbf{s'}|<r$, doing *b* instead of *a* does not make the situation worse, and improves it in some case.

Local dominance

Defining a metric on the profiles, we examine potential winners if we allow profiles which are within a certain distance r from a specific profile $\mathbf{s}$ – $L(\mathbf{s},r)$:

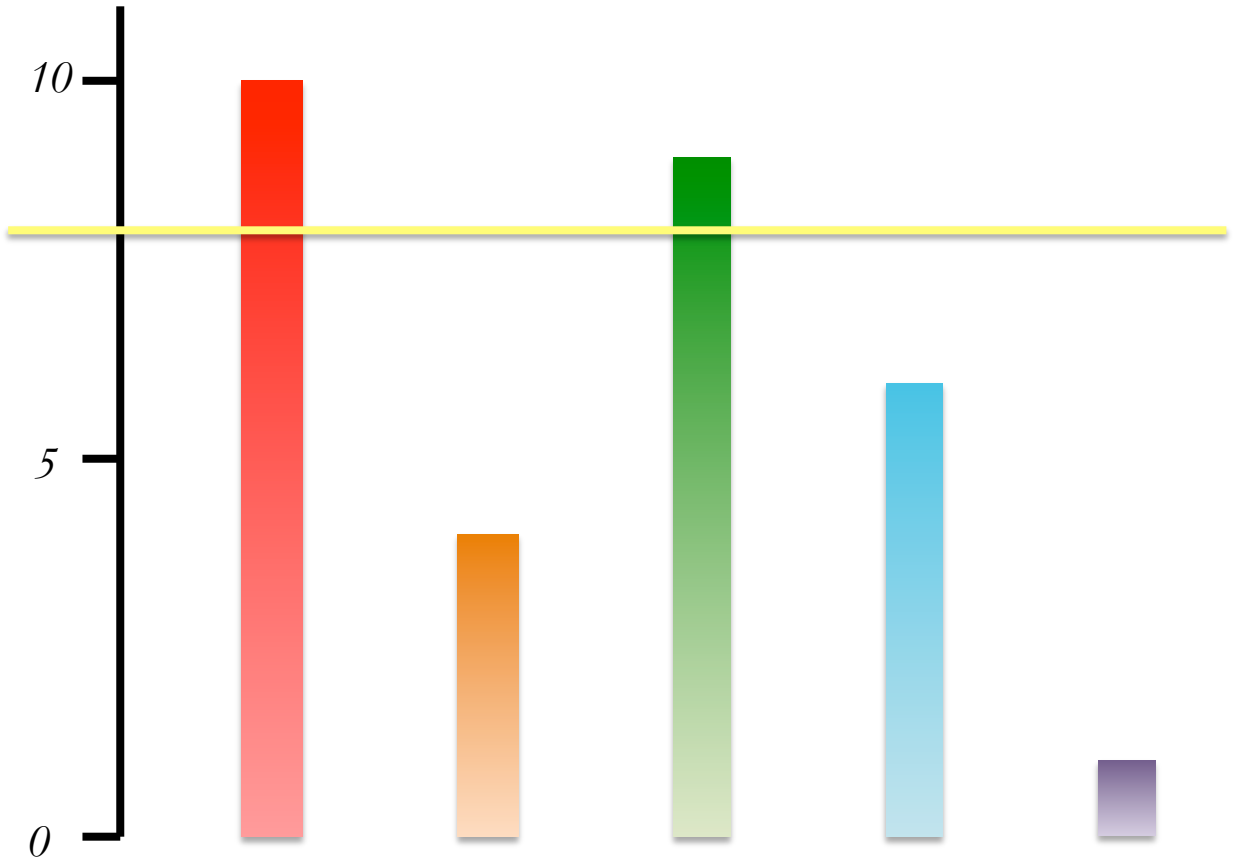
ℓ_1 additive: at most r voters can move

ℓ_∞ additive: every candidate can add/loose r voters.

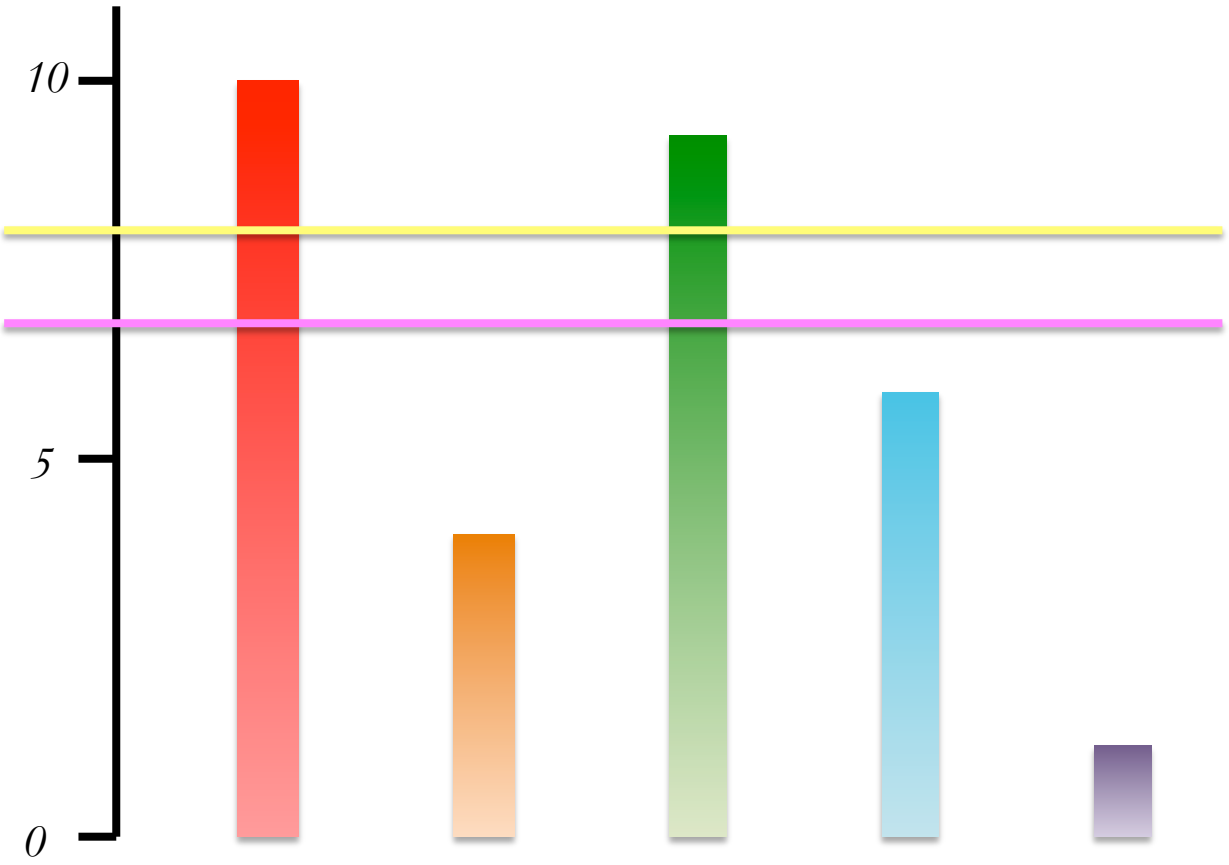
ℓ_1 multiplicative: at most rn voters can move

ℓ_∞ multiplicative: every candidate can add/loose rn voters.

Radius of uncertainty



Radius of uncertainty and of helplessness

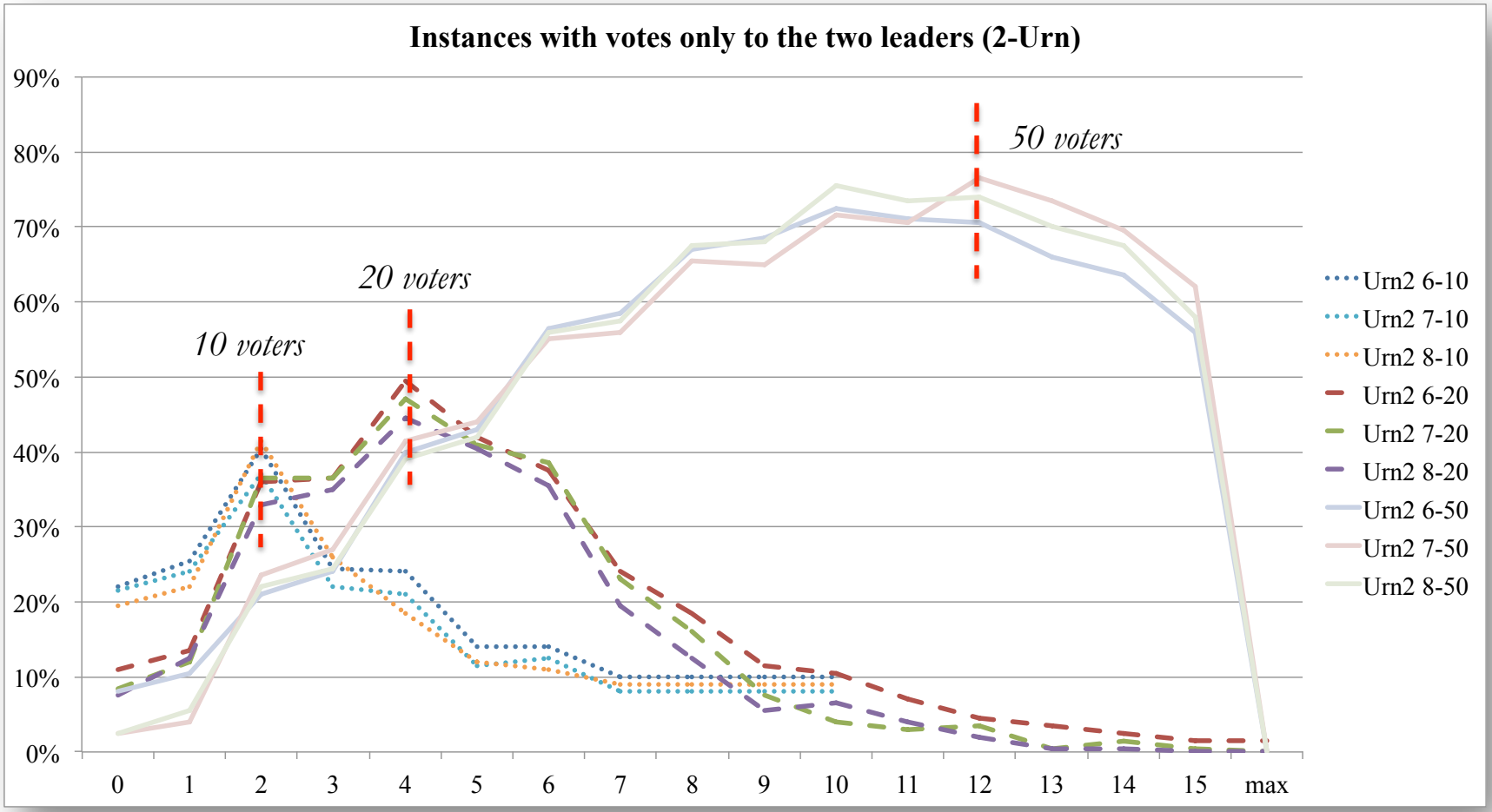


Does this model converge?

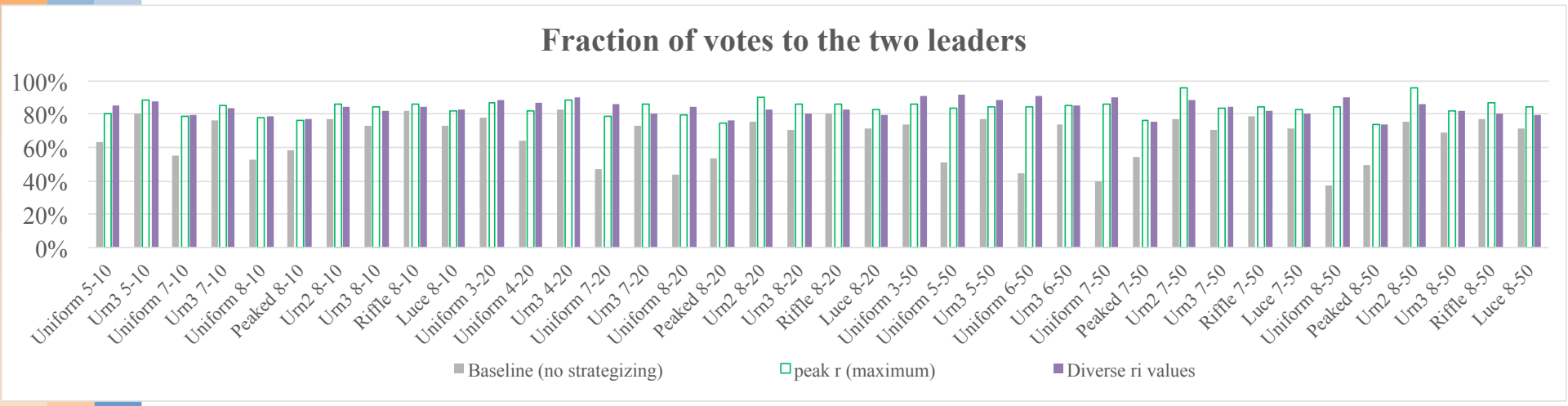
When starting from a truthful state, and all voters are from the same kind ("basic" / truth-biased / lazy-biased) with the same radius, it will always converge to a stable state.

Empirically, so do all other cases.

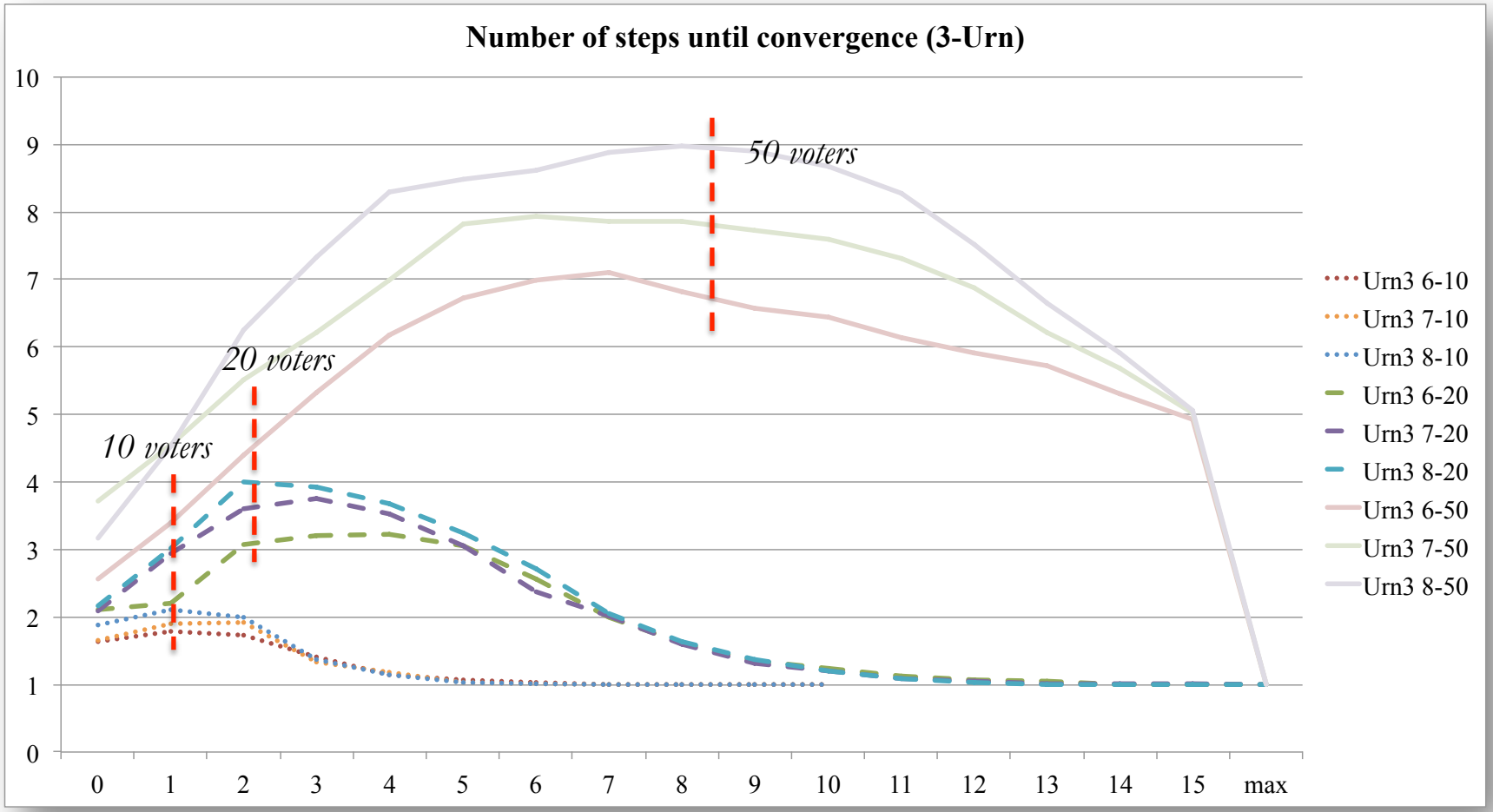
Properties Duverger law



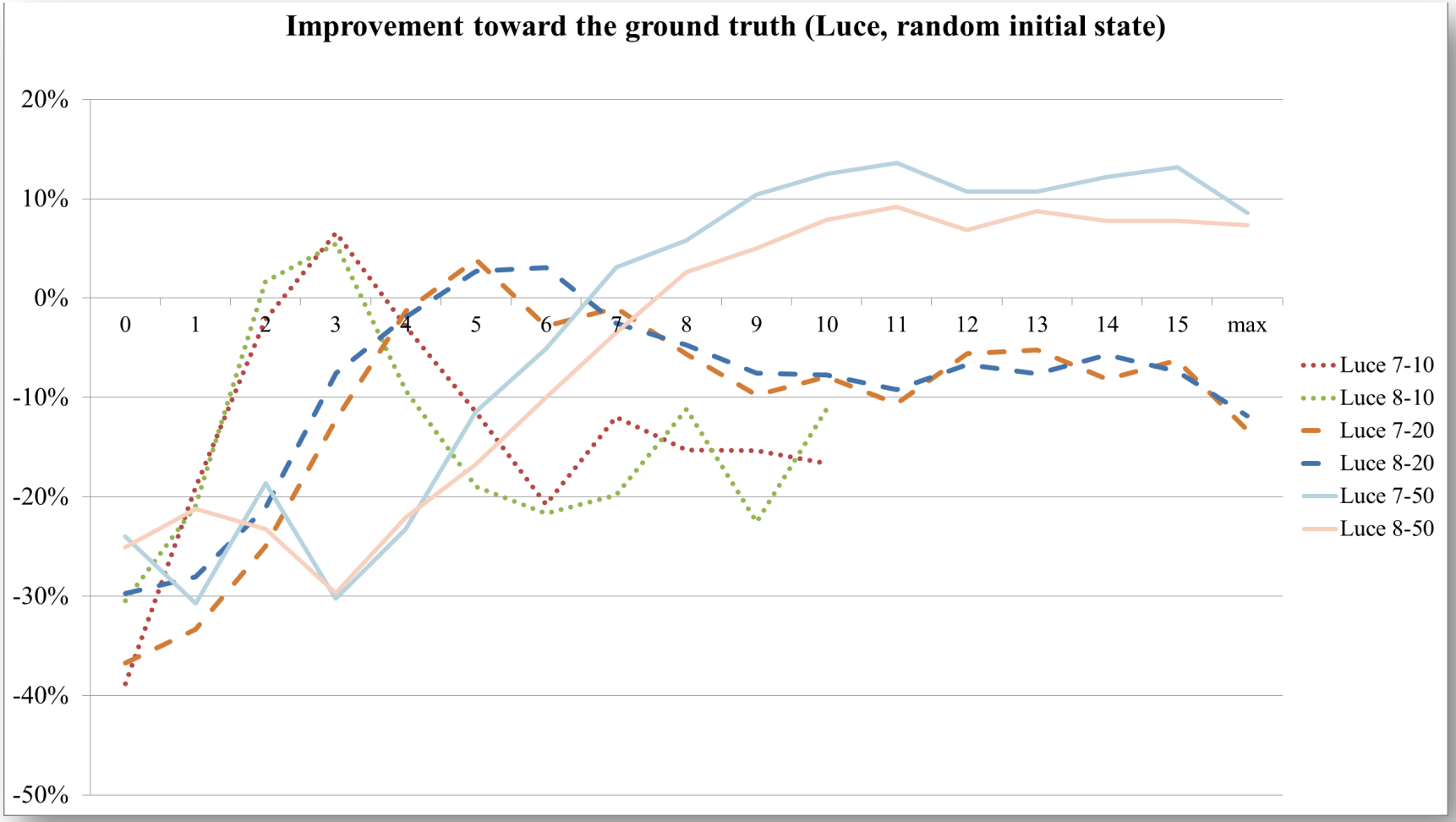
Properties Duverger law



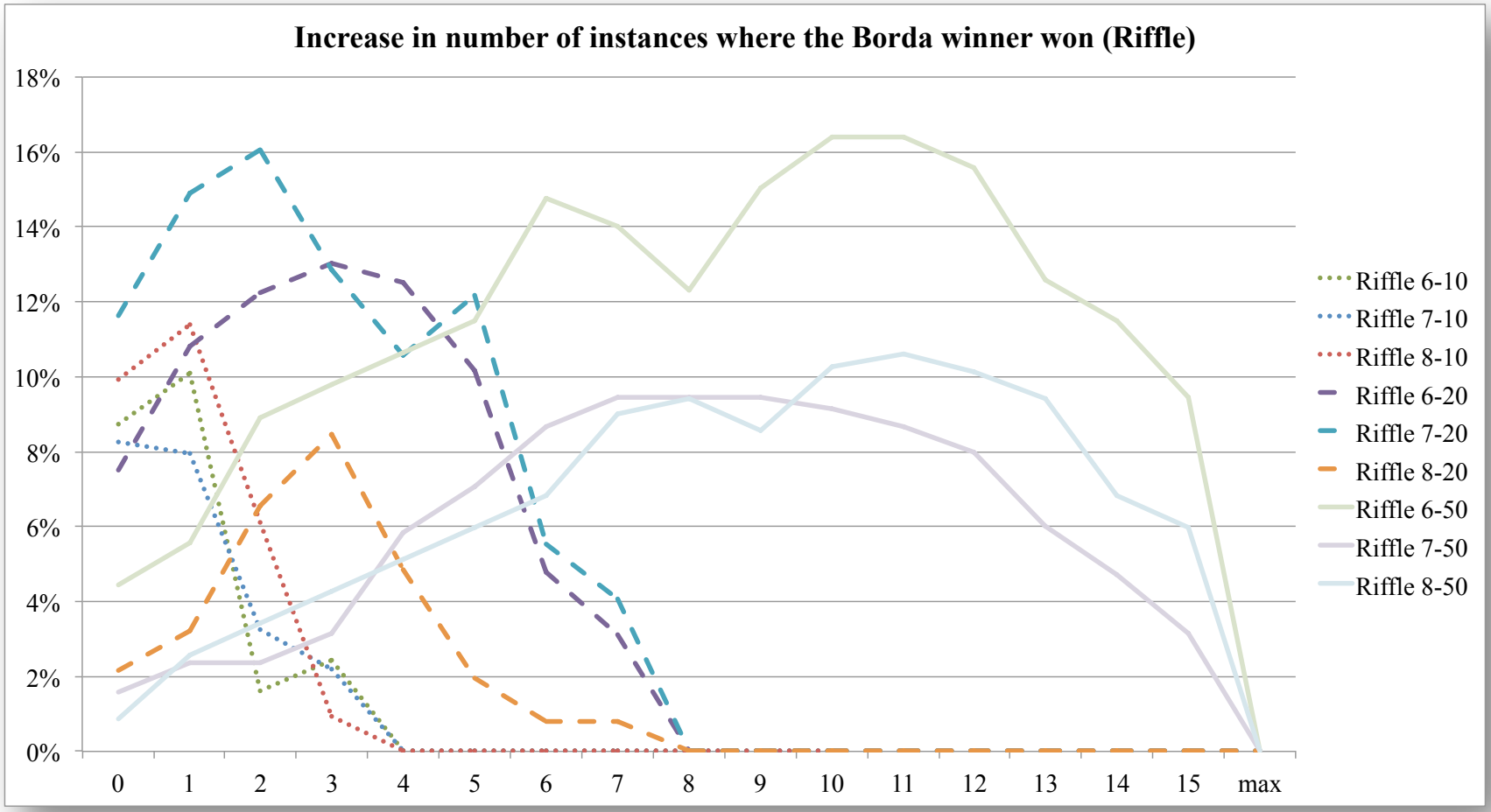
Properties Convergence speed



Properties ground truth (Placket-Luce)



Properties Borda score



What next?

More voting rules!
(less scoring rules?)

Exploring the local dominance model?

More empirical work

Non-concurrency of iterative process

Non myopic players?

Thanks for listening!

They couldn't decide either (from "The Muppet Movie")