A Mildly Personal Summary of COMSOC 2010
Düsseldorf, September 2010

Critical MAS, October 2010
Overview: COMSOC stats

- 92 participants (vs. 80 in Liverpool 2008).
- 39 papers accepted (out of 57 submissions).
- 5 invited talks.
- 7 “tutorial” talks.
- Düsseldorf: a city with more than 580,000 people - very little life detected on site…
Toby Walsh prodded the COMSOC community to expand its use of empirical research. Some papers showed its usefulness:

Davies et al. proved a theorem (order doesn’t matter when allocating Borda votes) in order to use greedy algorithms based on bin-packing for unweighted Borda manipulation. One of them had a 99% success rate of finding optimum.

Gelain et al. attempt a “local search” algorithm for the stable marriage problem (and one of its NP-Hard variants - SMTI, with incomplete lists), and reach an $O(n \log(n))$ algorithm for finding stable marriage, and one with excellent results in not-extremely-incomplete-lists cases (more populated → better results).

Toby Walsh also called for the establishment of a set of benchmark voting data for testing algorithms, but no one was assigned to the project…
Exploring phase-transitions, as a tool for analyzing NP-hard problems. For example, in 3-SAT:
Empirical work can be used to validate existing results. In this case, veto voting rule, and the $O/\Omega/\Theta(\sqrt{n})$ manipulability result (Proccacia and Rosenschein 2007):
When looking at hardness the question arises - why is it fairly easy to determine larger $n$?

**Heuristic:** Assuming hard problems are when difference between manipulators’ weight and necessary amount of vetoes aren’t large, one can see using the Central Limit Theorem that the possibility if this happening is close to 0 and $n$ grows.
And looking at empirical results, unexpected results arise:

![Graphs](image)

**Figure 6:** Manipulation of an election where votes are highly correlated and the result is “hung”. We plot the probability that a coalition of $m$ agents can elect a chosen candidate. Vetoes of the manipulators are weighted and uniformly drawn from $[0, k]$, the other agents have all vetoed the candidate that the manipulators wish to win, and the sum of the weights of the manipulators is twice that of the non-manipulators.

**Figure 7:** The cost to decide if a hung election can be manipulated. We plot the cost for the CKK algorithm to decide if a coalition of $m$ agents can manipulate a veto election. Vetoes of the manipulators are weighted and uniformly drawn from $[0, k]$, the other agents have all vetoed the candidate that the manipulators wish to win, and the sum of the weights of the manipulators is twice that of the non-manipulators.

What does this tell us about the Veto rule? How will similar results affect our understanding of manipulation difficulty?
Topic II: Cake Cutting

We have a birthday cake:

Jim, the birthday boy, wants the text
Julie, his young sister, wants all the flowers
John, his best friend, is eyeing the blue flower
Jake, his dad, desires the word “happy”

How do we reconcile these demands?
Topic II: Cake Cutting

The Model:

The cake is modeled as the interval $[0,1]$. Each participant has a normalized utility function on this interval, indicating which parts he finds desirable (several technical requirements apply, such as having a piecewise continuous density function).

A cake cutting is a function that allocates parts of $[0,1]$ to the players, and we usually require the parts to be a union of intervals.

Useful requirements:

**Proportional:** Each participant gets at least $1/n$ of the cake (by own valuation).

**Envy Free:** No participant wishes to get the allocation of someone else.

**Truthful:** No participant ever benefits from lying about his valuation.
Topic II: Cake Cutting

With two players, there is a simple algorithm to provide for all three requirements - “Cut and Choose”:

**Player 1** divides the cake into two parts.

**Player 2** selects the part he wants.

For more than 4 players, no similar algorithm is known. Not even one that satisfies Proportionality and Envy Free-nes.
A deterministic algorithm can satisfy all 3 criteria when dealing with value density functions which are constant (0 or value) (“piecewise uniform”). Thus we can ignore their value, but only care for the intervals:

```
Algorithm 1 \((V_1, \ldots, V_n)\)

1. \text{SubRoutine}\(\{1, \ldots, n\}, [0, 1], (V_1, \ldots, V_n)\)

\text{SubRoutine}\(S, X, V_1, \ldots, V_n\):
   1. If \(S = \emptyset\), return.
   2. Let \(S_{\text{min}} \in \arg\min_{S' \subseteq S} \text{avg}(S', X)\) (breaking ties arbitrarily).
   3. Let \(E_1, \ldots, E_n\) be an exact allocation with respect to \(S_{\text{min}}, X\) (breaking ties arbitrarily). For each \(i \in S_{\text{min}}\), set \(A_i = E_i\).
   4. \text{SubRoutine}\(S \setminus S_{\text{min}}, X \setminus D(S_{\text{min}}, X), (V_1, \ldots, V_n)\).
```
Existence of “exactness” (each participant gets only intervals he wants, and of the average length) proved using the Max-Flow Min-Cut theorem:

It’s easy to prove that the flow is the length of wanted intervals if there is no subset with smaller average, and using a lemma that an exact allocation exists iff the maximum flow is that of wanted intervals, the algorithm is well defined.
A random algorithm can satisfy all 3 criteria when dealing with value density functions which are piecewise linear:

**Algorithm 2 \((V_1, \ldots, V_n)\)**
1. Find a perfect partition \(X_1, \ldots, X_n\).
2. Draw a random permutation \(\pi\) over \(N\).
3. For each \(i \in N\), set \(A_i = X_{\pi(i)}\).

(perfect partition is one for which \(V_i(X_j) = 1/n\) for every player \(i\) and part \(j\) - does it exist?)

**Proportionality** and **Envy Free-ness** are trivial from partition perfectness, and **Truthfulness** is obvious, as the expectation from lying is \(1/n\), which is the same as the player will get from being truthful.
Achieving perfectness (who knew it was so simple?):

Every interval is divided to $2^n$ parts, and each player receives a part from the beginning and the end.

Looking at a player’s density function:

Each player get $1/n$ part of the interval (length & value wise).
We can define several easily understandable “consensus” classes, including elections which fit each one:

- **Strong unanimity**: Everyone has identical preference lists
- **Unanimity**: Everyone ranks the same candidate in first place
- **Majority**: More than half rank the same candidate in first place
- **Condorcet**: There is a Condorcet winner

Take a metric and a consensus class, and define the winner of each election as the winner of the closest election in the consensus class
Turns out, every voting rule can be defined with those consensus classes, using some metric (for example, all voting rules can be defined using **Strong Unanimity** by, essentially, defining every election is distance “1” from correct winner, “2” from others). But calculating metric is P if winner determination in P.

**Votewise rules**: Take \( d_C(x,y) \) - a metric on votes, and a norm on \( \mathbb{R}^n \), and our metric is \( d(E,E')=N(d_C(x_1,x'_1),...,d_C(x_n,x'_n)) \)

**Will this help?**
Every **scoring rule** can be defined using the **norm** $\ell_1$, and the **Unanimity** consensus class

**Plurality** may be defined by the **norm** $\ell_1$, and the **Strong unanimity** consensus class, but **Borda** can’t be defined by **Strong unanimity**.

**Simplified Bucklin** can be defined by the **norm** $\ell_\infty$, and the **Majority** consensus class.

**STV** can’t be rationalized by **Strong unanimity, Unanimity, or Majority** consensus classes.
Game structure is not regular voting:

Candidates = Voters

Electing k people

Voters mark who they approve of - no preference list (no list size)

Everyone wants to get elected

Consider it as a graph!
We want the candidates with the highest in-degree, obviously…

But is it strategyproof?

Consider any deterministic algorithm…

\[ k = 1? \]

Suppose the winner here is 1

So if 2 lies, then the winner here is 1, as well?

Approximation ratio = \( \infty \)

\[ k = 2? \ 3? \ … \ n-1? \]
2-RP: approximation ratio - 4

$k^{1/3}$-RP: approximation ratio - $1 + O(1/k^{1/3})$

But if want Group-Strategyproof - might as well choose random…
We want to satisfy as many people as possible by selecting a few “consensus” choices, that satisfy most people:

Locating parks throughout an area, satisfying most residents

Search results including major potential fields
When selecting a group of $k$ choices, using a scoring rule $\alpha$, we want a group $\Phi \subseteq A$ so that its score is maximized:

$$S_\alpha(\Phi, V) = \sum_{\ell \in N} \max_{a \in \Phi} \alpha_\ell(a)$$

When comparing it to top $k$ ranked by voting rule, Borda is a $1/2$-approximation of optimal score, while other rules may be worse than $1/k$-approximation.

A greedy algorithm is a $1-1/e$ approximation (proof by submodularity).
Expanding the framework, suppose every alternative $a$ has a fixed cost $t_a$ and a unit cost $u_a$. We now seek a function $\Phi : \mathbb{N} \rightarrow A$ that assigns agents to alternatives, and maximizes the score:

$$S_{\alpha}(\Phi, V) = \sum_{\ell \in N} \alpha_{\ell}(\Phi(\ell))$$

subject to a budget constraint, with the cost being:

$$C(\Phi) = \sum_{a \in A} 1[a \in \Phi(N)] \cdot t_a + \sum_{\ell \in N} u_{\Phi(\ell)}$$

A greedy algorithm (based on picking the best value for cost alternatives first) is $O(m^2n/\log(n))$ and on large datasets gives 97-98% of optimal score.
Fin

"Today: The collective unconscious..."

Thanks for listening!