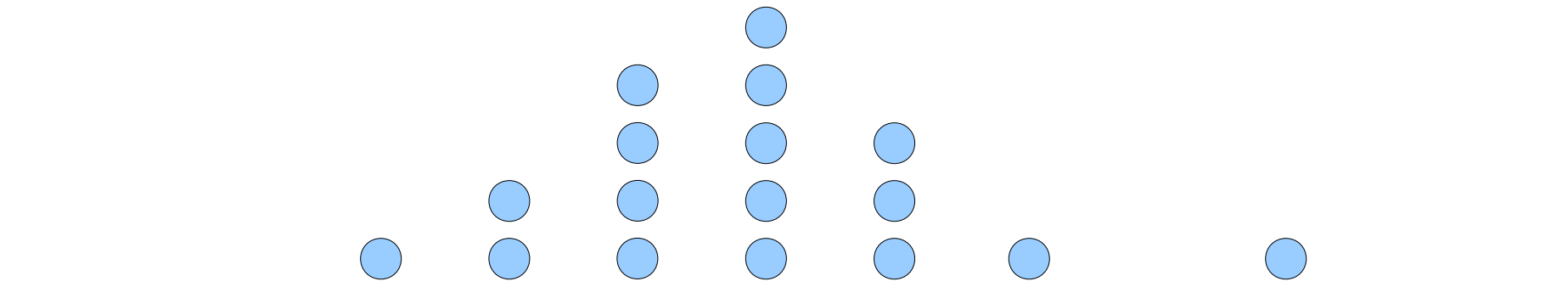


Experimental Approaches in Computer Science

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Hebrew University

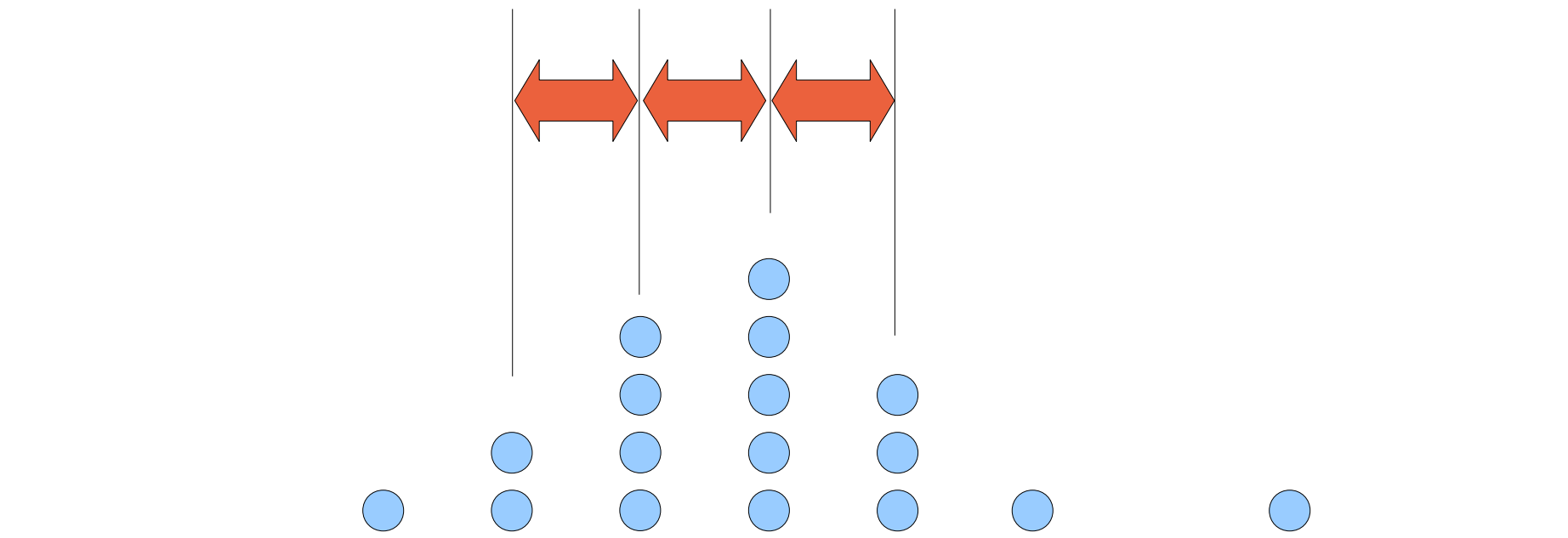
Lecture 3 – Measurements

Assume we start taking measurements of something:



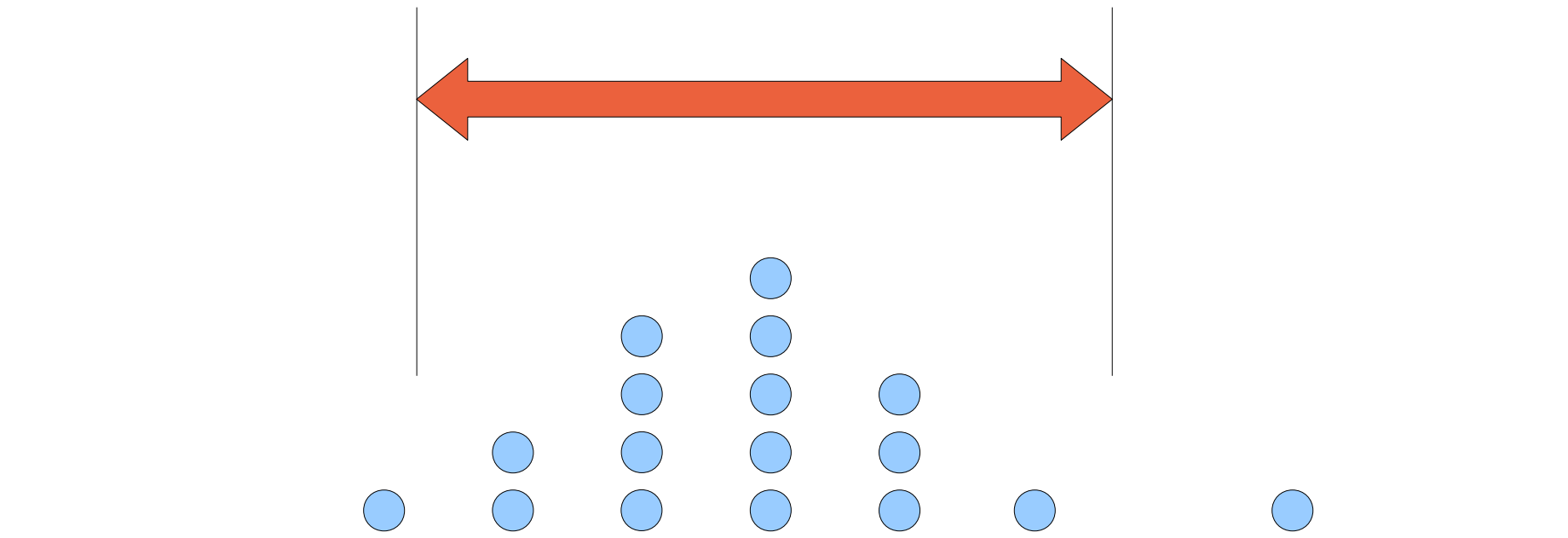
Such measurements are limited by the **resolution** of our measurement apparatus: the smallest difference between measurements

Example: measuring time in milliseconds



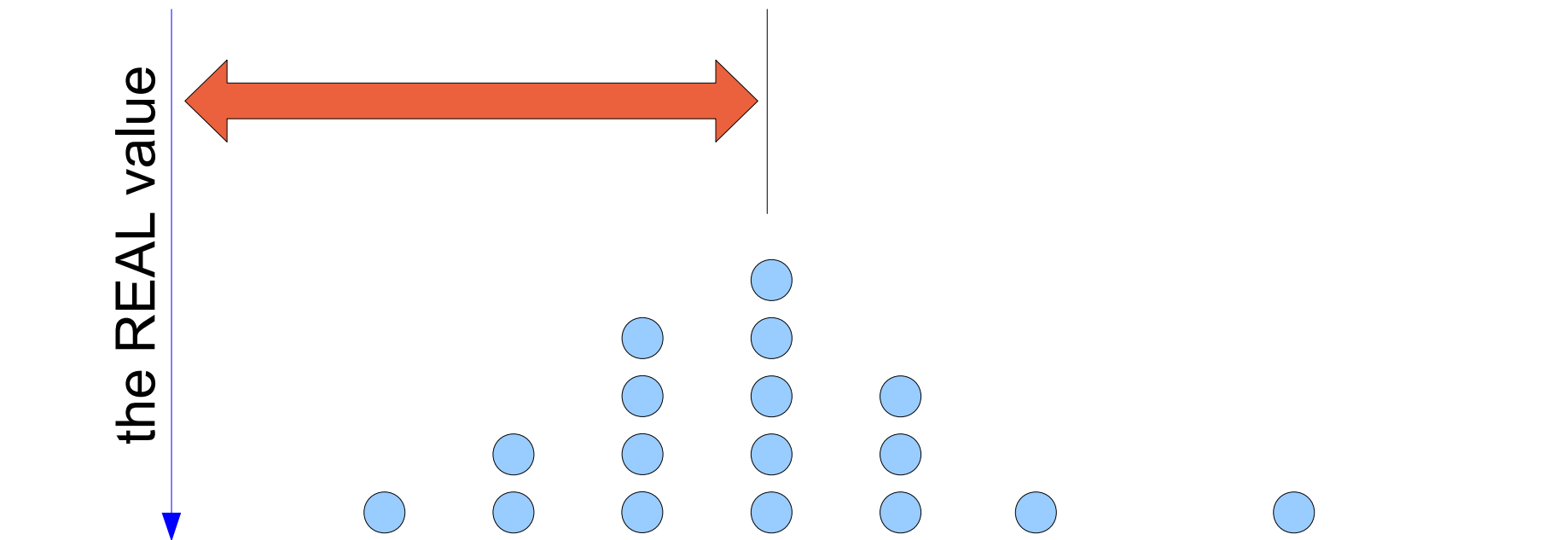
Another problem is the **uncertainty** in the measurements: if we repeat them, we get somewhat different results

This is caused by **random errors** that reflects the **(im)precision** of the measurement



A third issue is the **accuracy** of the measurement:
how far it is from the “real” value

This reflects the **systematic error** in the
measurement



Systematic errors are systematic – they always have the same effect

Example: when measuring the time to perform an action, the overhead of the measurement itself is added to the result

Part of the design should be to identify systematic errors and factor them out

A special problem is warmup or hysteresis-type errors, where the outcome depends on history; for example, the first measurement could be different from subsequent ones

Random errors are random – they may have different effects

Example: when measuring the time to perform an action, the result may depend on cache state, interrupts from the network, and competition from other processes

This can be analyzed statistically

In extreme cases, interference leads to outliers that should be ignored

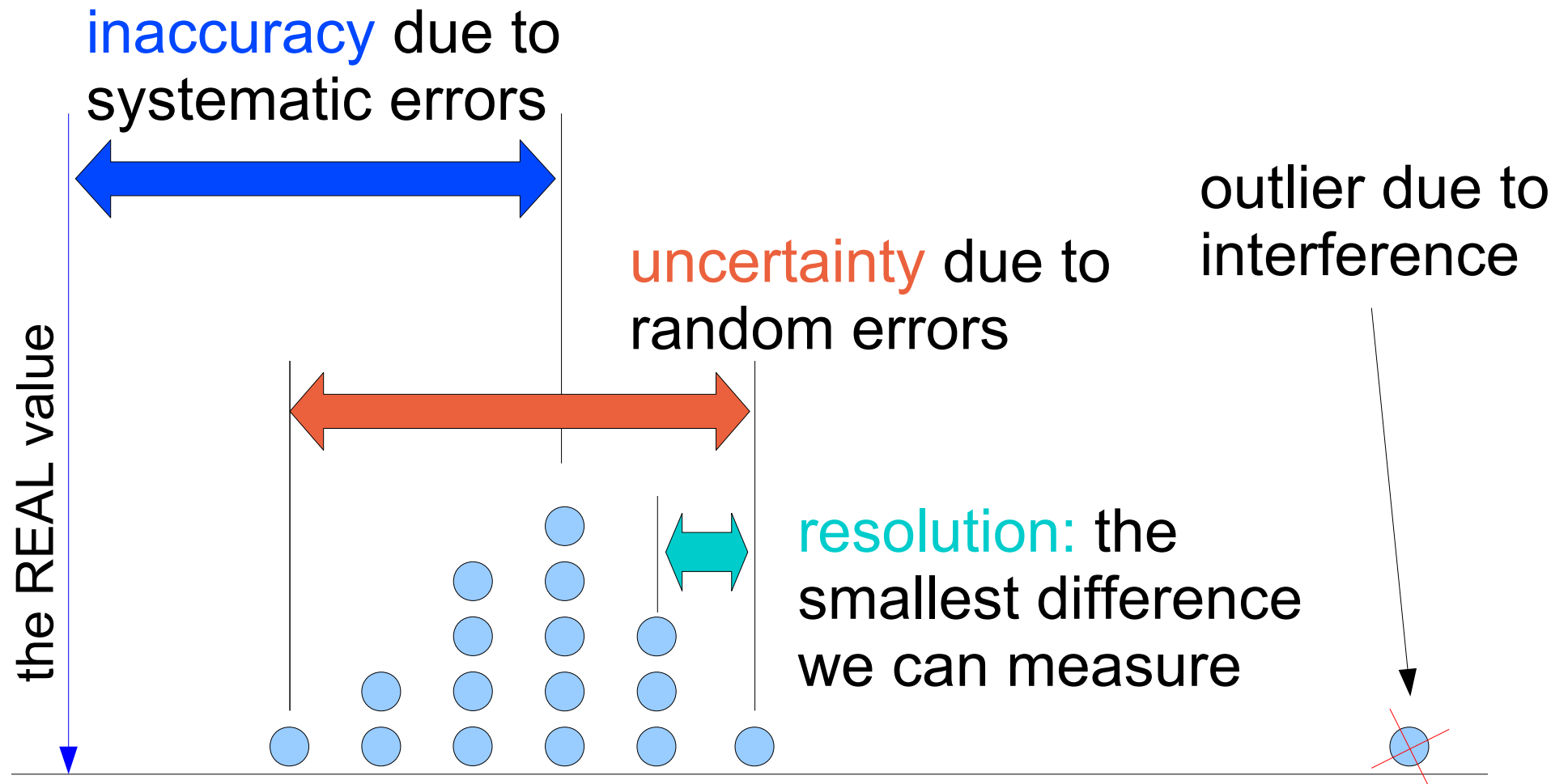
Model of random errors:

- Assume the error is a combination of multiple effects, each contributing $\pm\varepsilon$
- If we have n such effects, the outcome will have a binomial distribution

$$p(X + k \varepsilon) = \binom{n}{\frac{n+k}{2}} \frac{1}{2^n}$$

- For large n , this can be approximated by a normal distribution
- Calculation of confidence intervals is based on assumption of a normal distribution

Summary:



Measuring Time

A computer (specifically, Intel PC) has three time sources:

- Timer interrupts
- Time stamp counter
- Time server and NTP protocol

Timer interrupts:

- Cause a CPU interrupt, and running the clock interrupt handler
- Come at a certain frequency (e.g. 250 Hz) from an external timer
- The system counts how many interrupts (called ticks or jiffies) have occurred
- Reasonably accurate measure of wallclock time
- Typical resolution of several milliseconds (maximal resolution is microseconds)

Time stamp counter:

- A special register that counts the cycles since the machine was booted
- Can be read by a special assembler instruction
- Rate depends on the CPU clock rate
 - Can drift e.g. when temperature rises
 - Can change due to power considerations, especially on laptops (reduce speed to save energy)
- Nanosecond (cycle) resolution, but need many cycles to take a reading

NTP (Network Time Protocol)

- Most networks have a designated NTP server
- The NTP server gets time from some standard source
- All nodes in the network synchronize with the NTP server
- Effect is to update the system time
 - Can lead to a jump in time
 - Jump can also be backwards
 - Alternative is to change the time gradually

Linux `gettimeofday()`

- Upon each clock interrupt
 - Update notion of real time as per the external source
 - Synchronize with the time stamp counter
- When called, read the current time stamp counter and extrapolate from the previous clock interrupt
- Combines different timing sources
- Report result in microsecond resolution (API)

Making a Measurement

The framework:

- Measurement of some computer activity or operation
- Done from user level
- With no specialized tools

A simple measurement:

```
t1 = gettimeofday();
```

operation being measured

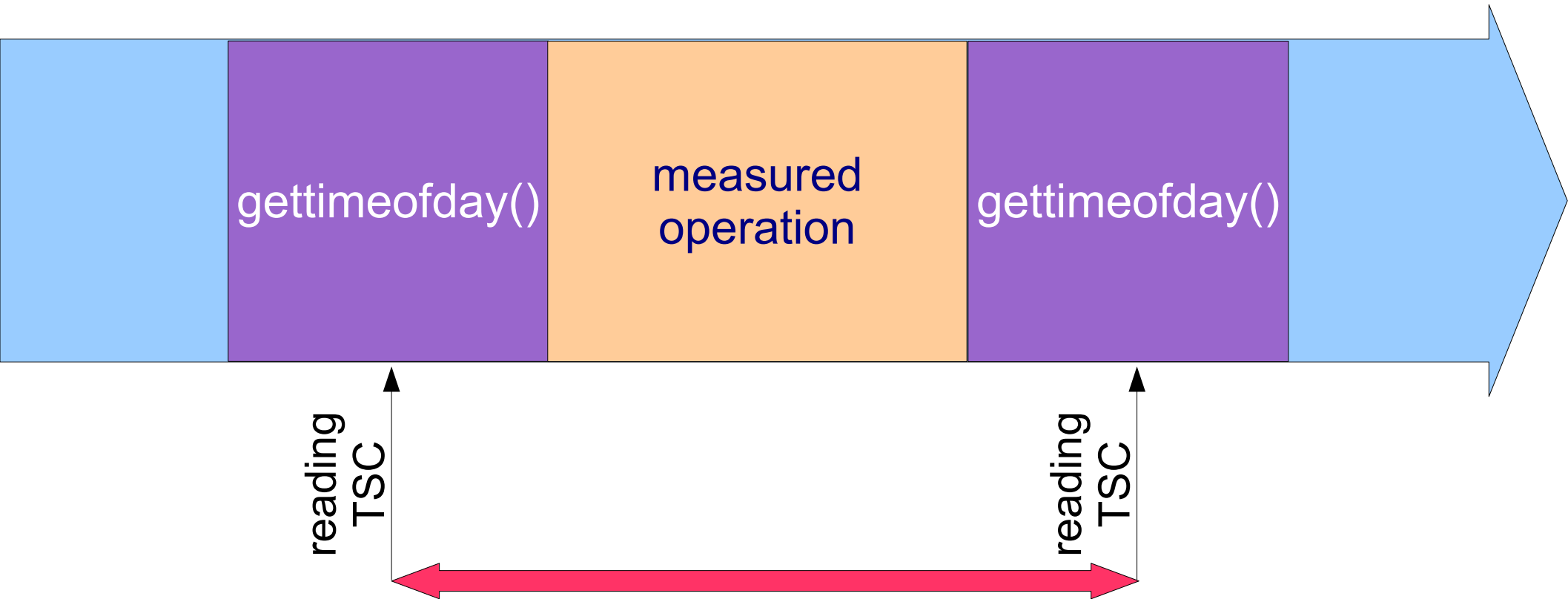
```
t2 = gettimeofday();
```

```
print "time was ", t2 - t1, "\n";
```

Potential problems:

- Inaccuracy due to measurement overhead

measurement overhead:



measured value includes operation + one
gettimeofday()

Thus one gettimeofday() should be subtracted

A simple measurement:

```
t1 = gettimeofday();
```

operation being measured

```
t2 = gettimeofday();
```

```
print "time was ", t2 - t1, "\n";
```

Potential problems:

- Inaccuracy due to measurement overhead
- Operation is shorter than our measurement resolution (get t1 = t2 so time = 0)
- Operation is short relative to random errors (so actually get a random time)

Solution 1: iterate making multiple measurements

```
for (i=0 ; i<N ; i++) {  
    t1 = gettimeofday();  
    operation being measured  
    t2 = gettimeofday();  
    print "time was ", t2 - t1, "\n";  
}
```

- Resolution problem: printouts will be 0 or 1
- Random errors: printouts will fluctuate wildly
- But in both cases the average (for large enough N) should be good

Solution 1a: buffer the results to reduce measurement interference

```
for (i=0 ; i<N ; i++) {  
    t1 = gettimeofday();  
    operation being measured  
    t2 = gettimeofday();  
    time[i] = t2 - t1;  
}  
  
print "average is ", avg(time[0..N-1]), "\n";
```

- Important because printing is a heavy operation that affects cache state and may cause a context switch

Solution 2: invert loop and measurement

```
t1 = gettimeofday();  
for (i=0 ; i<N ; i++) {  
    operation being measured  
}  
t2 = gettimeofday();  
print "average was ", (t2 - t1)/N, "\n";
```

- Measurement of $t2 - t1$ will now be stable and meaningful
- But need to subtract loop overhead (found by measuring an empty loop)

Problem: compiler optimizations

- Compiler may optimize away an empty loop
- Even if it has an operation in it like $j++$
 - If j is not subsequently used, why update it?
 - Even if it is, its value will grow just like that of the loop index
- Could work if the operation is on a global variable, because cross-procedural optimization is harder
- Still, need to check
 - e.g. verify that measured overhead depends on number of iterations

Solution 2a: reduce effect of overhead by unrolling

```
t1 = gettimeofday();
```

```
for (i=0 ; i<N/3 ; i++) {
```

```
    operation being measured
```

```
    operation being measured
```

```
    operation being measured
```

```
}
```

```
t2 = gettimeofday();
```

```
print "average was ", (t2 - t1)/N, "\n";
```

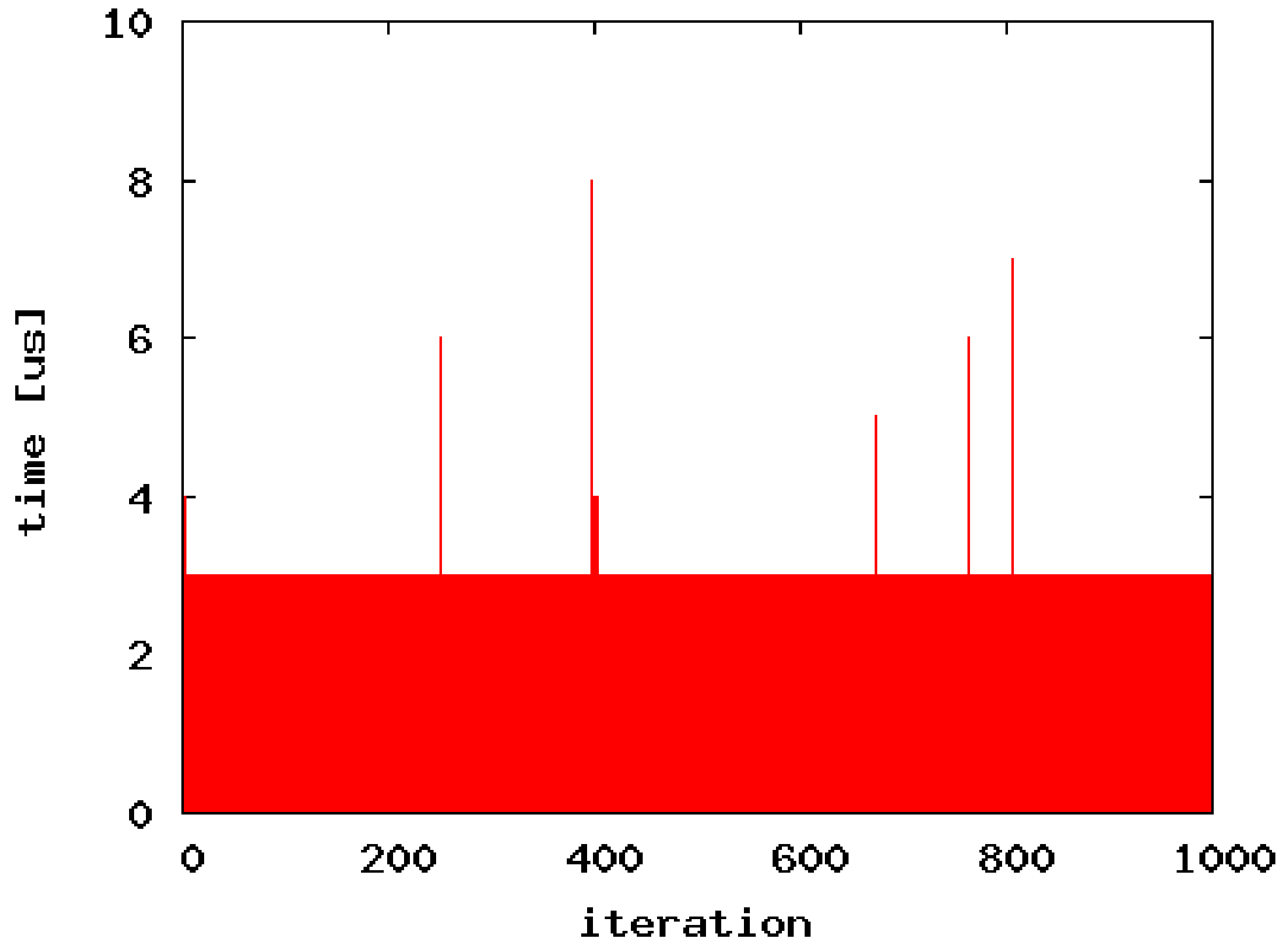
- Need to decide number of repetitions

In all iterative measurements, need to define **N**

- N should be big enough to pass resolution limit and average out random errors
- N should not be too big so as to reduce risk of large interferences from other activities in the system
 - Underlying assumption that measurement is short
 - Therefore completed within a single scheduling quantum without interference
 - If this is not the case, need to account for time spent doing other things
- But what if we're unlucky?

Example: time to write 100 bytes at the beginning of a file

outliers
are not
common,
but do
happen



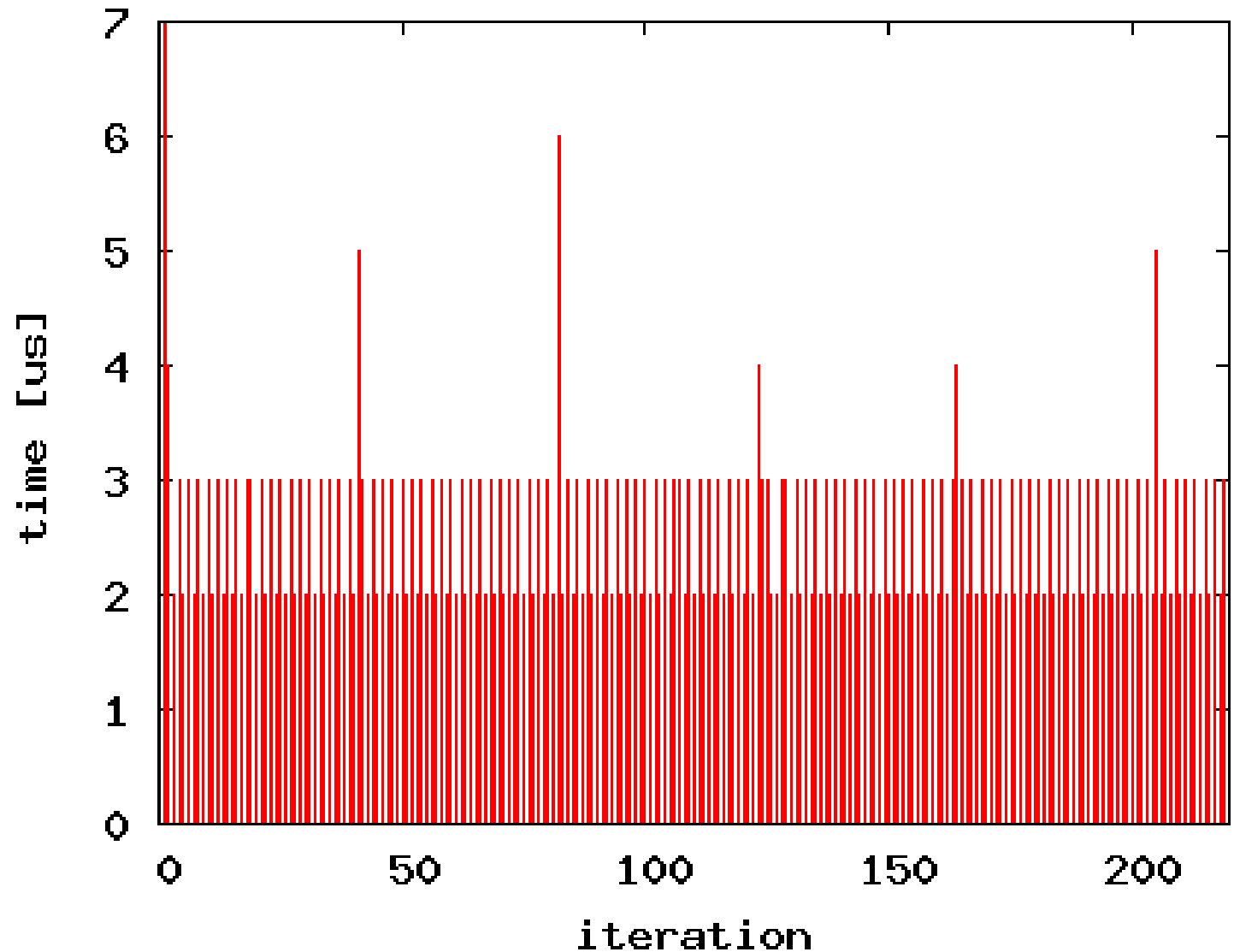
Solution 3: double loop

```
for (r=0 ; r<REP ; r++) {  
    t1 = gettimeofday();  
    for (i=0 ; i<N ; i++) {  
        operation being measured  
    }  
    t2 = gettimeofday();  
    print "average was ", (t2 - t1)/N, "\n";  
}
```

- Catch outliers in which interference has occurred

Outliers may also exhibit periodic behavior indicative of some system activity that needs to be understood

Example is writing successive blocks of 100 bytes to a file



Biggest problem:

Systematic errors that don't make sense

Need to look at the data

notice that something is wrong

identify it precisely

remove it from the analysis

Hard to do if sensitive to exact conditions

example
code:

```
for(i=0 ; i<=10 ; i++) {
    gettimeofday( &ts[i], 0 );
}
for(i=1 ; i<=10 ; i++) {
    printf("delta=%d\n",
        ts[i].tv_usec - ts[i-1].tv_usec);
}
```

```
gettimeofday( &ts[0], 0 );
gettimeofday( &ts[1], 0 );
...
gettimeofday( &ts[10], 0 );
```

```
for(i=1 ; i<=10 ; i++) {
    printf("delta=%d\n",
        ts[i].tv_usec - ts[i-1].tv_usec);
}
```

Possible
output:

delta=1
delta=1
delta=1
delta=1
delta=0
delta=1
delta=1
delta=1
delta=1

delta=2
delta=1
delta=1
delta=1
delta=1
delta=1
delta=1
delta=1
delta=1
delta=1

But
also:

delta=1
delta=1
delta=1
delta=1
delta=0
delta=1
delta=1
delta=1
delta=1
delta=1

delta=103
delta=1
delta=1
delta=33
delta=1
delta=1
delta=1
delta=1
delta=1
delta=1

so we seem to
have some
random
interference

But if we repeat this 10 times and average, a typical result is

avg delta = 1.20
avg delta = 1.00
avg delta = 0.70
avg delta = 0.90
avg delta = 1.00
avg delta = 1.00
avg delta = 0.90
avg delta = 1.00
avg delta = 1.10
avg delta = 1.00

(note: typical is not always...)

avg delta = 11.10
avg delta = 0.90
avg delta = 1.10
avg delta = 0.90
avg delta = 0.90
avg delta = 1.00
avg delta = 0.90
avg delta = 1.00
avg delta = 1.00
avg delta = 0.90

so there is actually some systematic problem at the beginning of the second batch of measurements

Sensitivity:

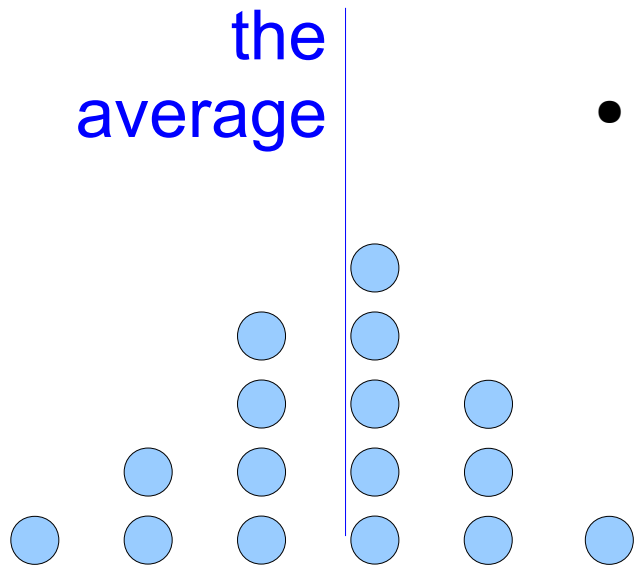
- This only happens when output is directed to the terminal; all is well if it is directed to a file
- This is related to printing the results of the first batch just before starting the second batch; all is well if all printing is done at the end

Calculating Confidence Intervals

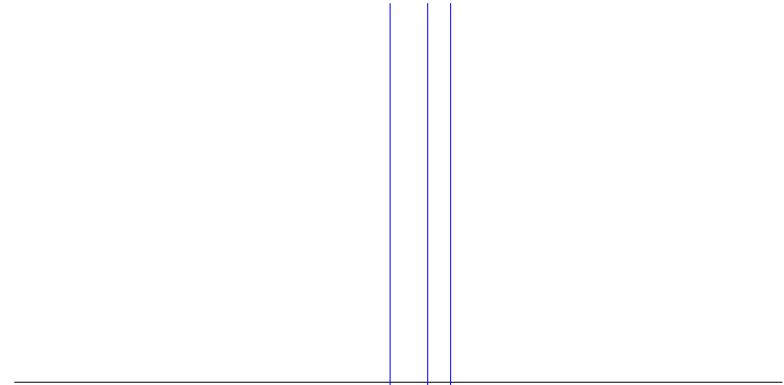
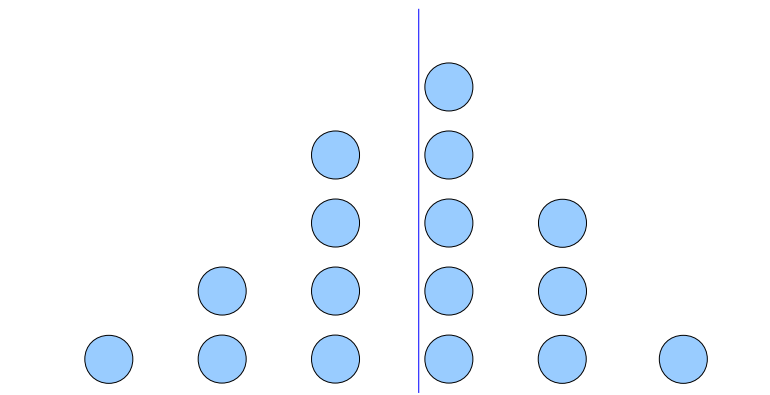
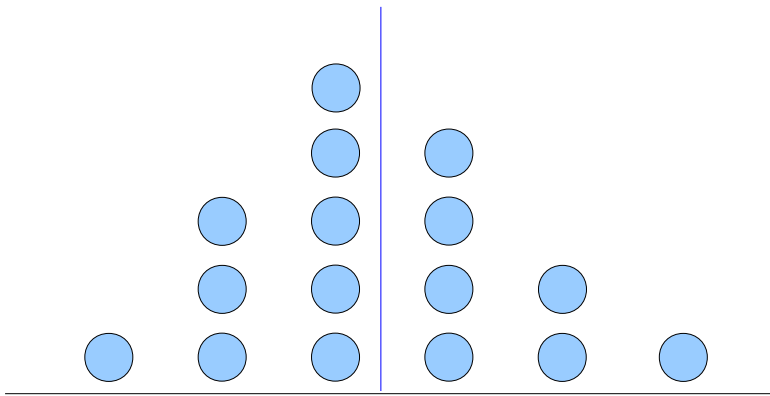
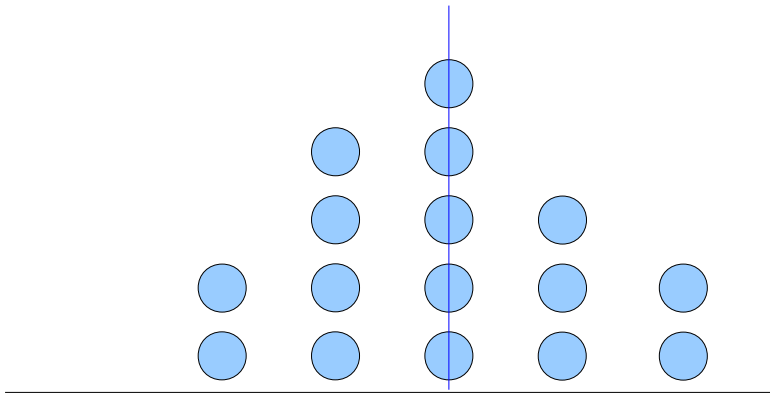
When we perform multiple measurements of the same thing, we can calculate confidence intervals

- 1) Assume measurements are samples from a (normal) distribution
- 2) Characterize the distribution's dispersion
- 3) Find the range that includes the desired mass of the probability density (e.g. 90%)

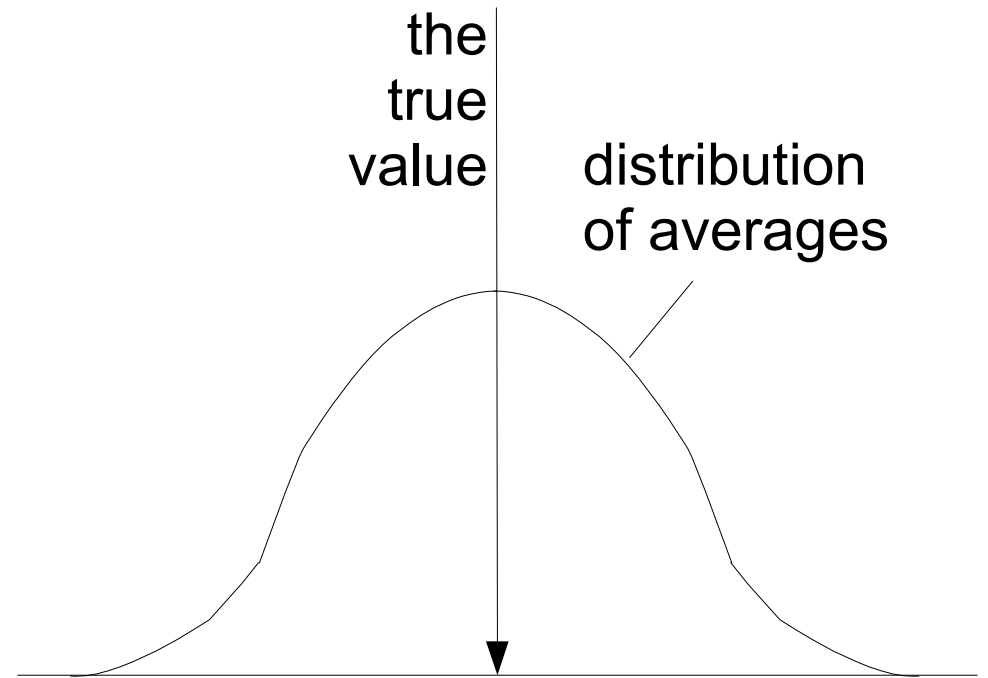
- Assume a set of measurements come from a normal distribution (real value + random error)
- This set has an average, which is an estimate of the real value
- If we repeat this with different samples, we will get a slightly different average
- So multiple samples from the base distribution induce a single sample from the distribution of averages



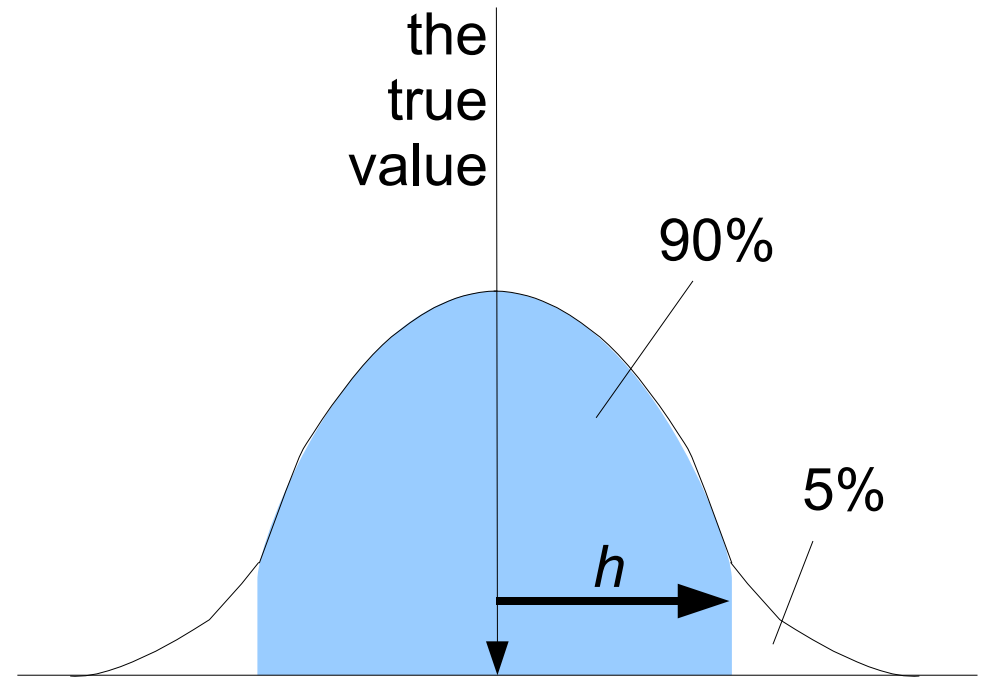
- multiple sets of samples induce multiple samples from the distribution of averages
- The distribution of averages is narrower than the base distribution
- So it gives a tighter estimate of the real value



- Assumption: the averages reflect a true value plus some random noise
- Thus the averages are distributed around the true value

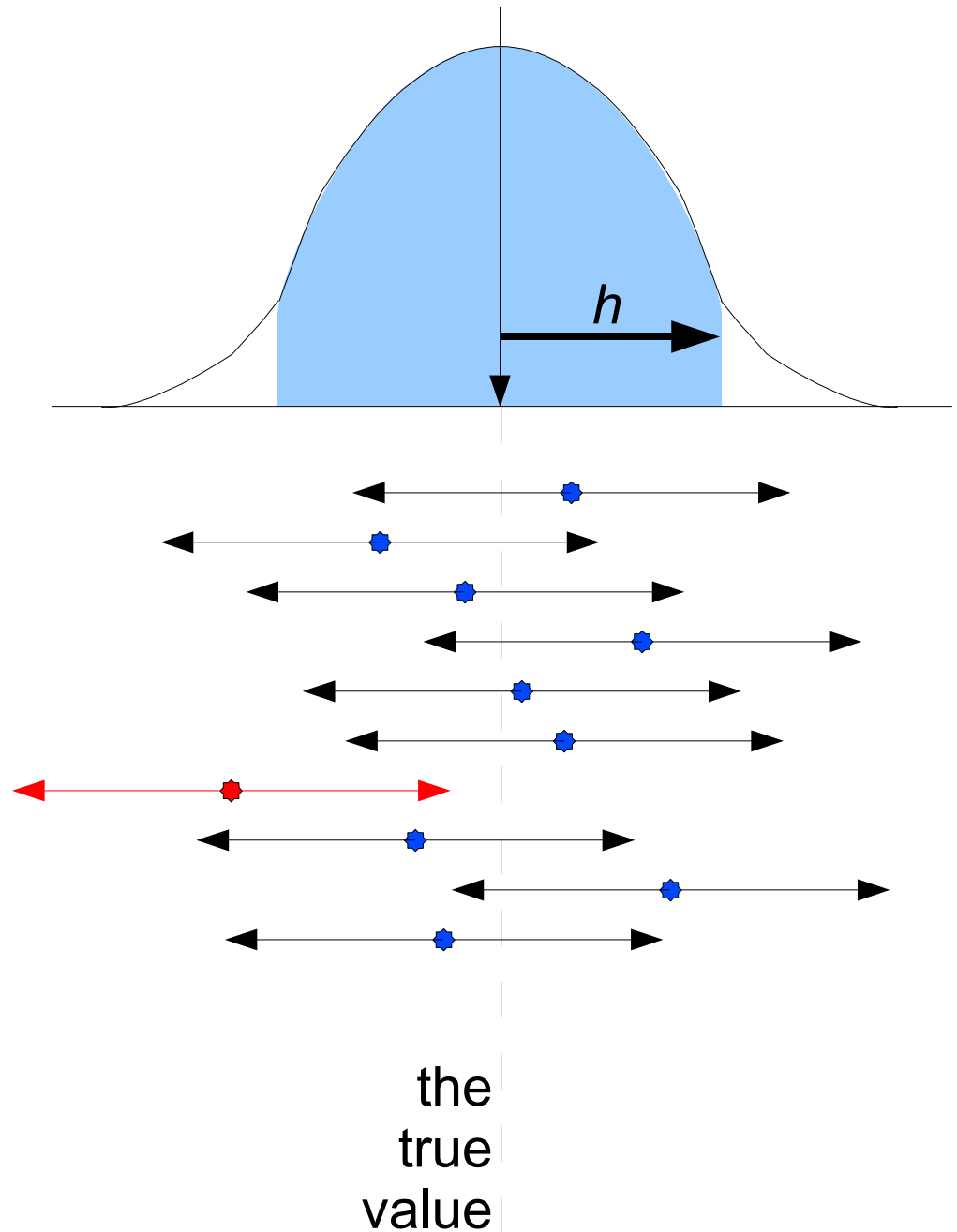


- Assumption: the averages reflect a true value plus some random noise
- Thus the averages are distributed around the true value
- Given the distribution, we can find the range h that is expected to contain 90% of the averages



This also works
the other way:
for 90% of the
averages, the
true value is
within h

*So the range
average $\pm h$ has
probability 0.9 to
include the real
value*



- Let μ denote the real mean of the base distribution
- Let \bar{x} denote the average of n samples
- If the base distribution is normal, then the averages have a t distribution
- Let α denote the acceptable uncertainty (implying that the level of confidence is $1-\alpha$)
- Define the half-width to be

$$h = t_{n-1, 1-\alpha/2} S_{\bar{x}}$$

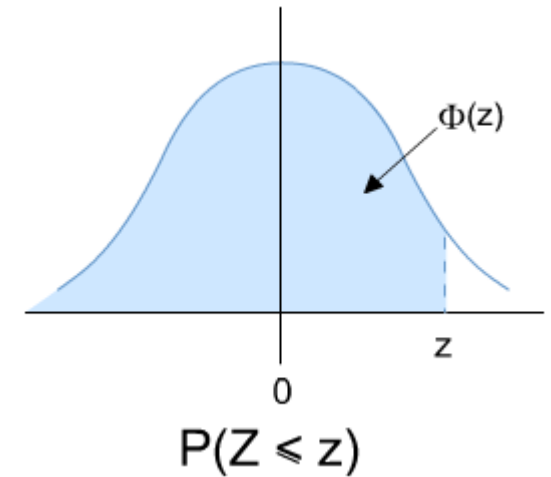
- Then

$$p(|\bar{x} - \mu| < h) = 1 - \alpha$$

The half width:

$$h = t_{n-1, 1-\alpha/2} S_{\bar{x}}$$

- $t_{n-1, 1-\alpha/2}$ comes from tables
 - $n-1$ degrees of freedom
 - n is the number of samples
 - for large n approximated by the normal $Z_{1-\alpha/2}$
- $S_{\bar{x}}$ is the standard deviation of the averages
 - assuming the base samples are *independent*, this can be calculated as s/\sqrt{n} (where s is the standard deviation of the base samples)
 - with more samples, the distribution of the averages becomes narrower



The confidence interval:

$$p(|\bar{x} - \mu| < h) = 1 - \alpha$$

- with a certainty of $1 - \alpha$, the distance between a sample of the average \bar{x} and the true mean μ is less than h
- If we repeat this many times, and each time we draw a segment of $\pm h$ around \bar{x} , then in $1 - \alpha$ of the cases this segment will include μ

Assumptions:

- The base samples come from a normal distribution

If not, but have a finite variance, the averages will still be normal, but this requires a larger n

- Base samples are independent

If not, maybe using larger batches will reduce the correlation between them

Note: **first** clean the data, **then** compute confidence intervals

- Remove outliers that indicate interference
 - Mechanized approach: remove top and bottom measurements
 - Better approach: look at the data!
 - Usually, outliers are only bigger (interference cannot reduce the measured time)
- Remove history/warmup effects