Targil 7:

Image Warping

2D Geometric Transformations

From Last Class JPEG-2000 V.S. JPEG visual comparison

JPEG: DCT (blocks of 8\times8)
JPEG 2000: Wavelets (global transformation)
Noticeable difference in high level of compression

JPEG-2000 V.S. JPEG

Compression at 0.25 b/p by means of (a) JPEG (b) JPEG-2000

JPEG-2000 V.S. JPEG

Compression at 0.2 b/p by means of (a) JPEG (b) JPEG-2000

JPEG-2000 V.S. JPEG

Reconstructed images compressed at 0.125 bpp by means of (a) JPEG and (b) JPEG2000

JPEG-2000 V.S. JPEG

JPEG 2000 (1.83 KB)
Original (979 KB)
JPEG (6.21 KB)
Image Warping

image filtering: change range of image
\[ g(x) = T(f(x)) \]

image warping: change domain of image
\[ g(x) = f(T(x)) \]

Parametric (global) warping

Examples of parametric warps:
- translation
- rotation
- aspect
- affine
- perspective
- cylindrical

Scaling

Scaling a coordinate means multiplying each of its components by a scalar.
Uniform scaling means this scalar is the same for all components.
Scaling

Non-uniform scaling: different scalars per component:

$$\begin{align*}
X \times 2, \\
Y \times 0.5
\end{align*}$$

Scaling operation:

$$
\begin{align*}
x' &= ax \\
y' &= by
\end{align*}
$$

Or, in matrix form:

$$
\begin{bmatrix}
x' \\
y'
\end{bmatrix} =
\begin{bmatrix}
a & 0 \\
0 & b
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
$$

What's inverse of $S$?

2-D Rotation

$$
\begin{align*}
x' &= x \cos(\theta) - y \sin(\theta) \\
y' &= x \sin(\theta) + y \cos(\theta)
\end{align*}
$$

2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

2D Identity?

$$
\begin{align*}
x' &= x \\
y' &= y
\end{align*}
$$

2D Scale around (0,0)?

$$
\begin{align*}
x' &= sx \times x \\
y' &= sy \times y
\end{align*}
$$

2D Rotation

This is easy to capture in matrix form:

$$
\begin{bmatrix}
x' \\
y'
\end{bmatrix} =
\begin{bmatrix}
\cos(\theta) & -\sin(\theta) \\
\sin(\theta) & \cos(\theta)
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
$$

Even though $\sin(\theta)$ and $\cos(\theta)$ are nonlinear functions of $\theta$,
- $x'$ is a linear combination of $x$ and $y$
- $y'$ is a linear combination of $x$ and $y$

What is the inverse transformation?

- Rotation by $-\theta$
- For rotation matrices, $\det(R) = 1$, so $R^{-1} = R^T$
What types of transformations can be represented with a 2x2 matrix?

2D Rotate around (0,0):
\[ x' = \cos \theta x - \sin \theta y \]
\[ y' = \sin \theta x + \cos \theta y \]

2D Shear:
\[ x' = x + sh_y y \]
\[ y' = sh_x x + y \]

2D Mirror about Y axis:
\[ x' = -x \]
\[ y' = y \]

2D Mirror over (0,0):
\[ x' = -x \]
\[ y' = -y \]

2D Translation:
\[ x' = x + t_x \]
\[ y' = y + t_y \]

Only linear 2D transformations can be represented with a 2x2 matrix.

Linear transformations are combinations of...
- Scale
- Rotation
- Shear
- Mirror

Properties of linear transformations:
- Origin maps to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

\[ \begin{bmatrix} x' \ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \]
Homogeneous Coordinates

- Represent coordinates in 2 dimensions with a 3-vector:
  \[
  \begin{pmatrix}
  x \\
  y \\
  1
  \end{pmatrix}
  \]

Multiplying by a scalar does not change our point!

\[
\begin{pmatrix}
2x \\
2y \\
2
\end{pmatrix}
= \begin{pmatrix}
3.5x \\
3.5y \\
3.5
\end{pmatrix}
= \begin{pmatrix}
x \\
y \\
3.5
\end{pmatrix}
\]

are all equivalent to:

\[
\begin{pmatrix}
x \\
y \\
1
\end{pmatrix}
\]

Example of translation

Translation

\[
\begin{pmatrix}
x' \\
y' \\
1
\end{pmatrix}
= \begin{pmatrix}
x \\
y \\
1
\end{pmatrix}
\]

Q: How can we represent translation as a 3x3 matrix?

\[
x' = x + t_x \\
y' = y + t_y
\]

A: Using the rightmost column:

\[
\begin{pmatrix}
1 & 0 & t_x \\
0 & 1 & t_y \\
0 & 0 & 1
\end{pmatrix}
\]

Translation

Homogeneous Coordinates

Add a 3rd coordinate to every 2D point:

- \((x, y, w)\) represents a point at location \((x/w, y/w)\)
- \((0, 0, 0)\) is not allowed

Convenient coordinate system to represent many useful transformations

(2,1,1) or (4,2,2) or (6,3,3)

Basic 2D Transformations

Basic 2D transformations as 3x3 matrices

- Translate
  \[
  \begin{pmatrix}
x' \\
y' \\
1
\end{pmatrix}
= \begin{pmatrix}1 & 0 & t_x \\
0 & 1 & t_y \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
1
\end{pmatrix}
\]

- Scale
  \[
  \begin{pmatrix}
x' \\
y' \\
1
\end{pmatrix}
= \begin{pmatrix}a & 0 & 0 \\
0 & b & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
1
\end{pmatrix}
\]

- Rotate
  \[
  \begin{pmatrix}
x' \\
y' \\
1
\end{pmatrix}
= \begin{pmatrix}\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
1
\end{pmatrix}
\]

- Shear
  \[
  \begin{pmatrix}
x' \\
y' \\
1
\end{pmatrix}
= \begin{pmatrix}1 & ab & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
1
\end{pmatrix}
\]

Affine Transformations

Affine transformations are combinations of ...

- Linear transformations, and
- Translations

Properties of affine transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition
Affine transformations

When do we meet them?
- When the camera is scanning a plane in parallel, combining rotation, zoom.

Camera moving plane

Projective Transformations

Projective transformations ...
- Affine transformations, and
- Projective warps

Properties of projective transformations:
- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Ratios are not preserved
- Closed under composition

Matrix Composition

Transformations can be combined by matrix multiplication

\[
\begin{bmatrix}
  x' \\
  y' \\
  w'
\end{bmatrix}
= \begin{bmatrix}
  1 & 0 & ts_x & \cos \Theta & -\sin \Theta & 0 & 0 & 0 & 0 & 0 \\
  0 & 1 & ts_y & \sin \Theta & \cos \Theta & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  w
\end{bmatrix}
\]

\[p' = T(t_x, t_y) \circ R(\Theta) \circ S(s_x, s_y) \circ p\]

Projective Transformations

Central projection map points on one plane to points on another plane

Matrix Composition

Matrices are a convenient and efficient way to represent a sequence of transformations
- General purpose representation
- Hardware matrix multiply

\[p' = (T \circ R \circ S \circ p)\]
\[p' = (T \circ R \circ S) \circ p\]
Matrix Composition

Be aware: order of transformations matters
– Matrix multiplication is not commutative

\[ p' = T \ast R \ast S \ast p \]