Tirgul 5
Image Pyramids

Pyramids

\[ N^2 + \frac{1}{4} N^2 + \frac{1}{16} N^2 + \cdots = \frac{1}{3} N^2 \]

Efficient Visual Search

Motion Computation

Browsing in image Databases
**Applications for Pyramids**

- Detection and Search
- Motion Computations
- Browsing in Image Databases

**Image Resizing**

**Shrink:**
1. Blur
   - Convolve with a 3*3, or 5*5 or...
2. Sub-sample
   - Select every 2nd pixel in every 2nd row

**Expand:**
1. Zero Padding
2. Blur

**Blur Kernel for Pyramids**

Weights: $(a, b, c, b, a)$

Conditions:
- $c > b > a$
- $2a + 2b + c = 1$
- $c + 2a = 2b$

Commonly Used - Binomial Coefficients

1  1  1
1  4  6  4  1
1  6  15  20  15  6  1

**Blur & Sub-sample** *(Reduce)*

**Blur & Sub-sample**
Zero-Pad & Blur

Zero-Pad & Blur

Zero-Pad & Blur

Zero-Pad & Blur

Zero-Pad & Blur

Boundaries

Mirror on last pixel. \( ? = 2 \)
Mirror after last pixel. \( ? = 0 \)
Duplicate last pixel. \( ? = 0 \)
Gaussian Pyramid

\[ G_0 \rightarrow \text{Reduce}(G_{n-1}) \]
\[ G_1 \rightarrow \text{Reduce}(G_1) \]
\[ G_2 \rightarrow \text{Reduce}(G_2) \]
\[ G_3 \rightarrow \text{Reduce}(G_3) \]

2D Picture: Reduce Rows, Reduce Columns

Laplacian Pyramid

\[ L_0 \rightarrow G_0 \]
\[ L_1 \rightarrow \text{Expand}(G_1) \]
\[ L_2 \rightarrow \text{Expand}(G_2) \]
\[ L_3 \rightarrow \text{Expand}(G_3) \]
\[ L_4 \rightarrow \text{Expand}(G_4) \]

\[ L_n + L_{n-1} = \text{Expand}(L_n) + L_{n-1} = \]
\[ = \text{Expand}(G_n) + (G_{n-1} - \text{Expand}(G_n)) = G_{n-1} \]

Pyramid Compression

- Build a Laplacian Pyramid
- Quantize pyramid values to 3-5 values
- Compress using Entropy Compression
  - (Huffman, Lempel-Ziv)
- Reconstruct normally
- Next Generation: Wavelet Compression

Optimal Quantization (reminder)

\[ q_i = \frac{\int z \cdot p(z)dz}{\int p(z)dz} \]
\[ z_i = q_{i-1} + \frac{q_i}{2} \]
Pyramid Compression

Multiresolution Spline

- Given two images $A$ and $B$
- Construct Laplacian Pyramid $L_a$ and $L_b$
- Create a third Laplacian Pyramid $L_c$ where for each level $l$
  \[ L_c(i, j) = \begin{cases} 
  & L_a(i, j) + L_b(i, j) / 2 & \text{if } i < \text{width}/2 \\
  & (L_a(i, j) + L_b(i, j)) / 2 & \text{if } i = \text{width}/2 \\
  & L_b(i, j) & \text{if } i > \text{width}/2 
  \end{cases} \]
- Sum all levels $L_c$ in to get the blended image

Pyramid Blending Example 1

Picture Merging with Spline

- For every Row:
  \[ C(i) = H_f(i-x)A(i) + H_r(i-x)B(i) \]

Image Merging with Laplacian Pyramids

- Given two images $A$ and $B$, and a mask $M$
- Construct Laplacian Pyramids $L_a$ and $L_b$
- Construct a Gaussian Pyramid $G_m$
- Create a third Laplacian Pyramid $L_c$ where for each level $l$
  \[ L_c(i, j) = G_m(i, j) L_a(i, j) + (1 - G_m(i, j)) L_b(i, j) \]
- Sum all levels $L_c$ in to get the blended image

Pyramid Blending Arbitrary Shape
Pyramid Blending Example 2