Degradation / Restoration Model

- Goal: Reconstruct an image that has been degraded by using a priori knowledge of the degradation phenomenon

\[ f(x,y) \Rightarrow [\text{Degradation Function } H] \Rightarrow g(x,y) \Rightarrow \hat{f}(x,y) \Rightarrow [\text{Restoration filters }] \Rightarrow \hat{f}(x,y) \]

\[ g(x,y) = h(x,y) \ast f(x,y) + \eta(x,y) \]

Tirgul 4: Image Restoration in the Frequency Domain

Example for Image Reconstruction (Tomography)

Inverse Filtering (Deconvolution Filters)

\[ \hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)} \]

- We need to have some information on the degradation \( H \) and on the noise \( N \)

Estimation by image observation

- Look for information in the image itself: search for small section of the image containing simple structure (edge, point)
  - High signal to noise ratio
  - Guess the image in that area \( \hat{f}_s \)

\[ \hat{f}_s(x,y) \ast h_s(x,y) = g_s(x,y) \]

\[ H_s(u,v) = \frac{G_s(u,v)}{F_s(u,v)} \]

Estimating the Degradation function

- We need to have some information on the degradation function
  - Observation
  - Experimentation
  - Mathematical modeling
Estimation by modeling

- Mathematical model of degradation can be for example atmosphere turbulence

\[ h(u,v) = e^{-k(u^2+v^2)} \]

Estimation by Experiment

- If we have the equipment used to acquire the degraded image we can obtain accurate estimation of the degradation
  - Obtain an impulse response of the degradation using the same system setting

Estimation by modeling: uniform motion blur

- Assuming the image was blurred by uniform linear motion between the image and the sensor during image acquisition
  - Simplification:
    - \( y(t) = 0 \)
    - \( x(t) = at/T \)
  - The degradation: convolution with \( \text{rect} \)
  - Fourier: \( \text{sinc} \)

Estimating White Noise Spectrum

- Assuming additive white noise, and given the linearity of Fourier transform, the transform of \( G \) will be sum of the transforms

\[ G(u,v) = H(u,v)F(u,v) + N(u,v) \]

Noise Models

- Principle source of noise – acquisition / transmission
- White noise:
  - Fourier spectrum of the noise is constant
- We assume that noise is independent in spatial coordinates and that it is not correlated with the image
Minimum Mean Square Error (Wiener) Filtering

\[ \hat{F}(u, v) = \frac{1}{H(u, v)} \left[ \frac{H(u, v)}{|H(u, v)|^2} \right] G(u, v) \]

K can be chosen interactively to yield best visible results

Examples

Image corrupted by motion blur and additive noise

Inverse filtering

Wiener Filtering

Modeling atmosphere turbulence

\[ h(u, v) = e^{-\lambda(u^2 + v^2)} \]

Inverse filtering

radially limited

Inverse filtering

Wiener filter

(K is chosen interactively)

Reconstruction Example: Computerized Tomography

• Goal: reconstruct 2D slices of 3D data given projection

• Computerized Tomography
  – X-Ray: propagating beam
  – Photons are lost from the beam as they are absorbed in the body
  – Try to reconstruct linear attenuation coefficient representing the material
Mathematical Formulation: Radon Transform

$$p_\phi(t) = \int_{rayAB} f(x, y) ds$$

All points on the line $AB$ satisfy the equation:

$$x\cos(\phi) + y\sin(\phi) = t$$

$$p_\phi(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(x\sin\phi + y\cos\phi - t) dxdy$$

Fourier Slice Theorem

The Inverse Radon

Fourier slice theorem:

$$S_0(\omega) = F(u, 0)$$

This is true for each angle.

To reconstruct the 2D images: perform inverse Fourier transform on collection of lines

Iterative Back Projection

- Start with initial guess of the image
- For each ray – perform summation on the guess and on the real image
- Spread the difference between guess and observation uniformly for all pixels along this ray
- This process is converging in many cases to the actual image

Algebraic reconstruction

- Solving linear equation system

$$\sum w_i f_j = p_i$$