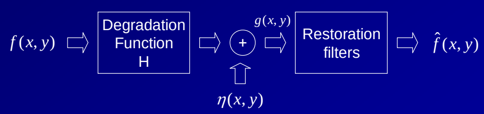
Degradation / Restoration Model

- Goal: Reconstruct an image that has been degraded by using a priori knowledge of the degradation phenomenon



$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

Tirgul 4: Image Restoration in the Frequency Domain

Example for Image Reconstruction (Tomography)

Inverse Filtering (Deconvolution Filters)

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

- We need to have some information on the degradation *H* and on the noise *N*

Degradation / Restoration Model

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

Equivalent frequency domain representation

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Estimation by image observation

- Look for information in the image itself : search for small section of the image containing simple structure (edge, point)
  - High signal to noise ratio
  - Guess the image in that area $\hat{f}_s$

$$\hat{f}_s(x,y) * h_s(x,y) = g_s(x,y)$$

$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

Estimating the Degradation function

- We need to have some information on the degradation function
  - Observation
  - Experimentation
  - Mathematical modeling

Estimation by modeling

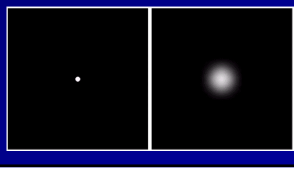
- Mathematical model of degradation can be for example atmosphere turbulence

$$h(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

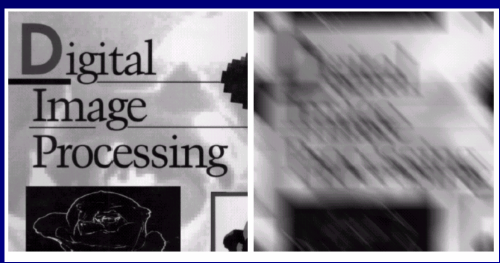
$$k = \begin{cases} 0.0025 \\ 0.001 \\ 0.00025 \end{cases}$$

Estimation by Experiment

- If we have the equipment used to acquire the degraded image we can obtain accurate estimation of the degradation
  - Obtain an impulse response of the degradation using the same system setting



Estimation by modeling: uniform motion blur



Estimation by modeling: uniform motion blur

- Assuming the image was blurred by uniform linear motion between the image and the sensor during image acquisition
  - Simplification:
    - *y(t) = 0*
    - *x(t) = at/T*
- The degradation : convolution with ***rect***
- Fourier : ***sinc***

Estimating White Noise Spectrum

- Assuming additive white noise, and given the linearity of Fourier transform, the transform of G will be sum of the transforms

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Noise Models

- Principle source of noise – acquisition / transmission
- White noise :
  - Fourier spectrum of the noise is constant
- We assume that noise is independent in spatial coordinates and that it is not correlated with the image

Minimum Mean Square Error (Wiener) Filtering

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v) / S_f(u,v)} \right] G(u,v)$$

K

K can be chosen interactively to yield best visible results

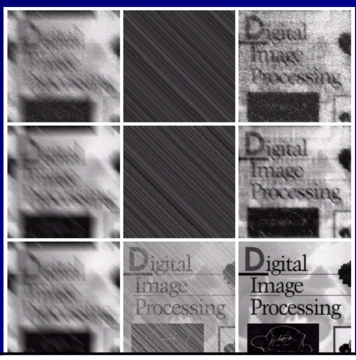
Minimum Mean Square Error (Wiener) Filtering

- Goal: have an inverse filter that is dealing with noise in better way than direct inverse filtering
  - Trying to minimize $e^2 = E\{(f - \hat{f})^2\}$

$$|H(u,v)|^2 - \text{power spectrum}$$
$$S_\eta(u,v) = |N(u,v)|^2$$
$$S_f(u,v) = |F(u,v)|^2$$

Examples

Image corrupted by motion blur and additive noise

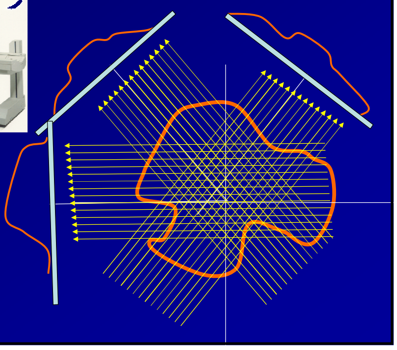


Examples

Modeling atmosphere turbulence $h(u,v) = e^{-k(u^2+v^2)^{5/6}}$

Inverse filtering — radially limited Inverse filtering — Wiener filter (K is chosen interactively)

Parallel projection



Reconstruction Example: Computerized Tomography

- Goal : reconstruct 2D slices of 3D data given projection
- Computerized Tomography
  - X-Ray : propagating beam
  - Photons are lost from the beam as they are absorbed in the body
  - Try to reconstruct linear attenuation coefficient representing the material

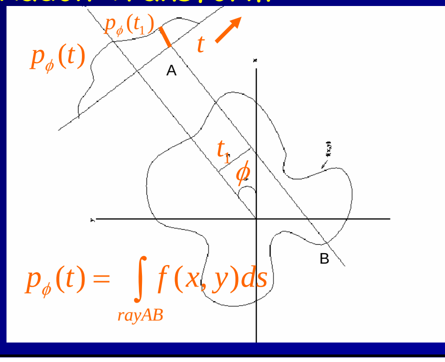
Radon Transform

$$p_\phi(t) = \int_{rayAB} f(x,y)ds$$

All points on the line AB satisfy the equation :
*x*cos(ϕ) + y*sin(ϕ) = t*

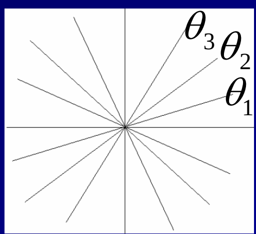
$$p_\phi(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(x\sin\phi + y\cos\phi - t)dxdy$$

Mathematical Formulation: Radon Transform



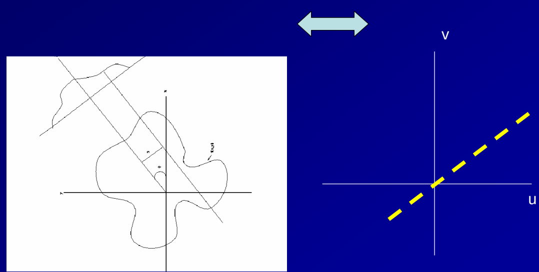
The Inverse Radon

Fourier slice theorem: $S_0(\omega) = F(u,0)$



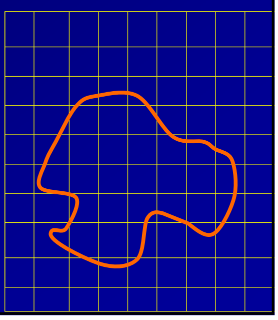
- *This is true for each angle.*
- *To reconstruct the 2D images: perform inverse Fourier transform on collection of lines*

Fourier Slice Theorem



Iterative Back Projection

- Start with initial guess of the image
- For each ray – perform summation on the guess and on the real image
- Spread the difference between guess and observation uniformly for all pixels along this ray
- This process is converging in many cases to the actual image



Algebraic reconstruction

- Solving linear equation system

$$\sum w_{ij} f_j = p_i$$