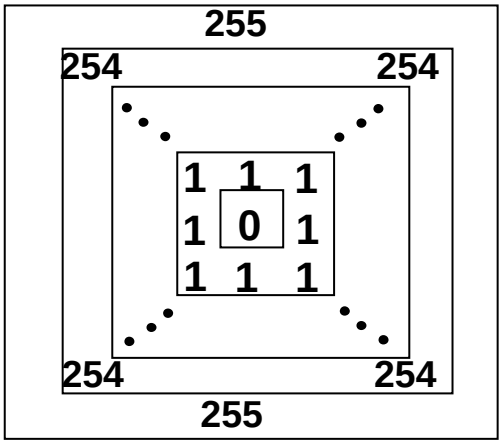
Image Processing: Theoretical Exercise #1

due: 29/11/05



1. We apply the following transformations (one after the other) on the above image :

   a. Histogram equalization to the range of 0..255.

   b. A simple binarization (Values 0.. 127 become 0, and the rest become 1).

   How will the resulting image look like ? How many zeros will be in this image ?

2. You are given the matrix *g* , obtained from the convolution: $g = f * \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

   How will you recover the original image *f* from *g* ? (The center of the convolution kernel corresponds to location (0,0). You can ignore the boundaries of the picture).

3. Prove the following properties of the Fourier Transform: (*f(x,y)* is the image, and *F(u,v)* is its Discrete Fourier Transform)

   a. Translation: $f(x-x_0, y-y_0) \Leftrightarrow F(u,v)e^{\frac{2\pi i(ux_0+vy_0)}{N}}$

   b. Symmetry (Assuming real images !): $F(u,v) = F^*(-u,-v)$

   c. Periodicity: $F(u,v) = F(u+N,v) = F(u,v+N)$

4. An image *f(x,y)* and it's Fourier Transform *F(u,v)* are given.

   a. What is the Fourier transform of the Laplacian of f(x,y), defined by

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

   b. Show that *F(0,0)* is real

   c. Show that |*F(0,0)*| is equal or bigger than any other |*F(u,v)*|

5. Given an Image *f(x,y)* in the size *NxN*, and and image *g(x,y)* in the size *MxM (M>N)* created by taking *f*, putting it in the center of *g* and padding with zeros the remaining pixels as illustrated below. What will be the differences between the Fourier Transform of *f* and *g*.