1. We apply the following transformations (one after the other) on the above image:
   a. Histogram equalization to the range of 0..255.
   b. A simple binarization (Values 0..127 become 0, and the rest become 1).

   How will the resulting image look like? How many zeros will be in this image?

2. You are given the matrix $g$, obtained from the convolution:

   $g = f * \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

   How will you recover the original image $f$ from $g$? (The center of the convolution kernel corresponds to location (0,0). You can ignore the boundaries of the picture).

3. Prove the following properties of the Fourier Transform: ($f(x,y)$ is the image, and $F(u,v)$ is its Discrete Fourier Transform)
   
   a. **Translation**: $f(x-x_0, y-y_0) \Leftrightarrow F(u,v) e^{-\frac{2\pi i (u x_0 + v y_0)}{N}}$
   
   b. **Symmetry (Assuming real images!)**: $F(u, v) = F^*(-u, -v)$
   
   c. **Periodicity**: $F(u,v) = F(u+N,v) = F(u,v+N)$
4. An image $f(x,y)$ and it’s Fourier Transform $F(u,v)$ are given.
   
   a. What is the Fourier transform of the Laplacian of $f(x,y)$, defined by
   
   $$
   \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}
   $$
   
   b. Show that $F(0,0)$ is real
   
   c. Show that $|F(0,0)|$ is equal or bigger than any other $|F(u,v)|$

5. Given an image $f(x,y)$ in the size $NxN$, and and image $g(x,y)$ in the size $MxM$ ($M>N$) created by taking $f$, putting it in the center of $g$ and padding with zeros the remaining pixels as illustrated below. What will be the differences between the Fourier Transform of $f$ and $g$. 

![Image](img)