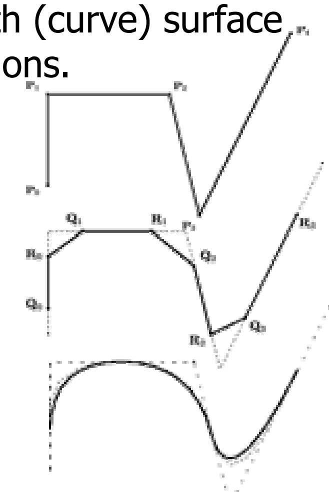
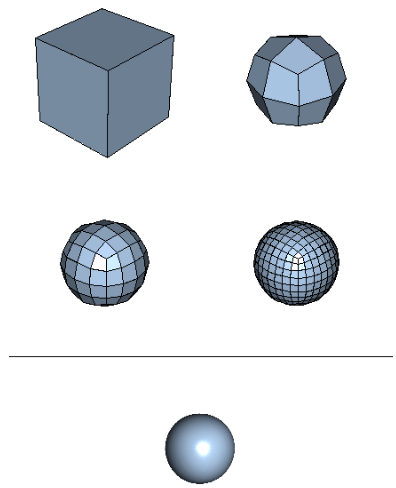
Computer Graphics Course 2005

Introduction to Subdivision Surfaces

Subdivision

- The process of creating a smooth (curve) surface by an (infinite) number of iterations.
- **Input**: polygonal control point
- **Process**: repeated refinements
- and averaging
- **Result**: smooth (curve) surface



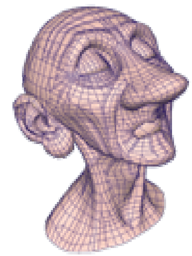


Why use subdivision ?

- Generates smooth surfaces from polygonal meshes of arbitral topology
- Efficient rendering
- Easy to animate
- Level of detail
- Compression
- Smoothing

Where subdivision was used?



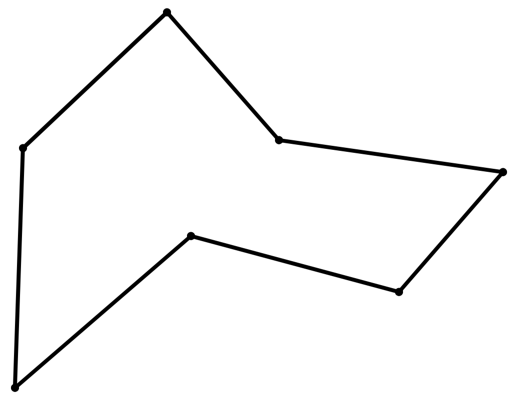



⌘Two main groups of schemes:

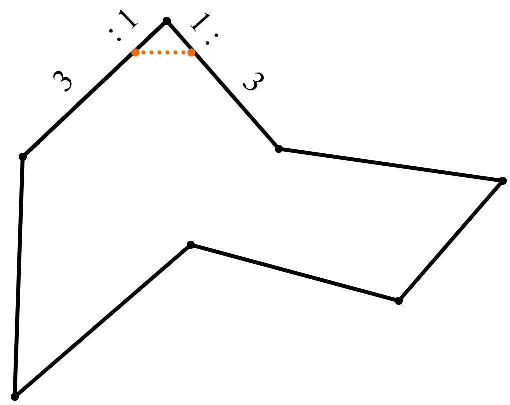
⌘Approximating - original vertices are moved

⌘Interpolating – original vertices are unaffected

Corner Cutting



Corner Cutting



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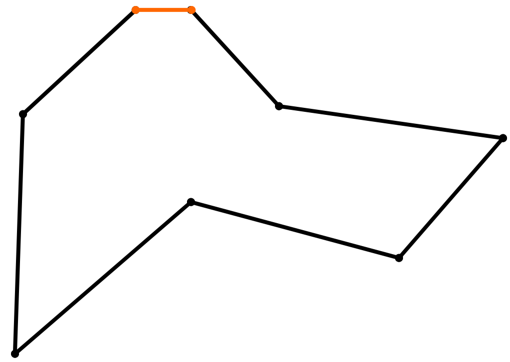

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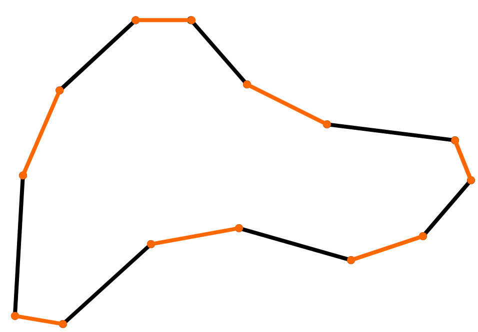

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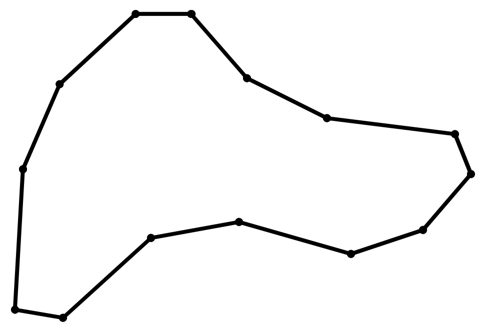

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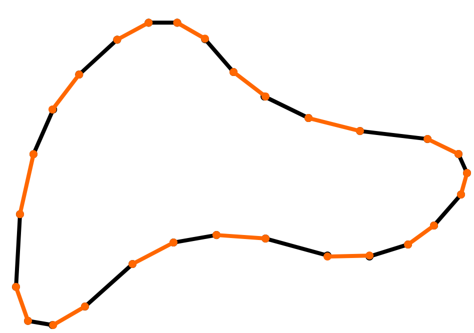

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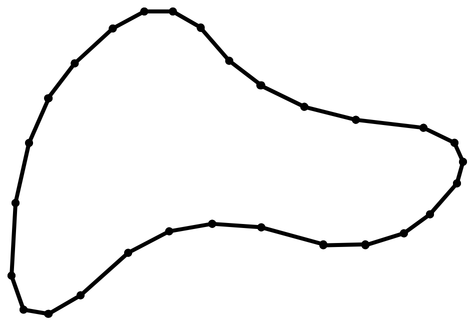

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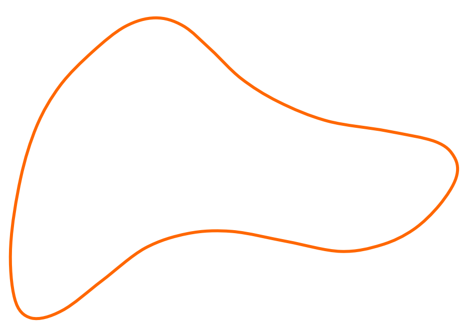

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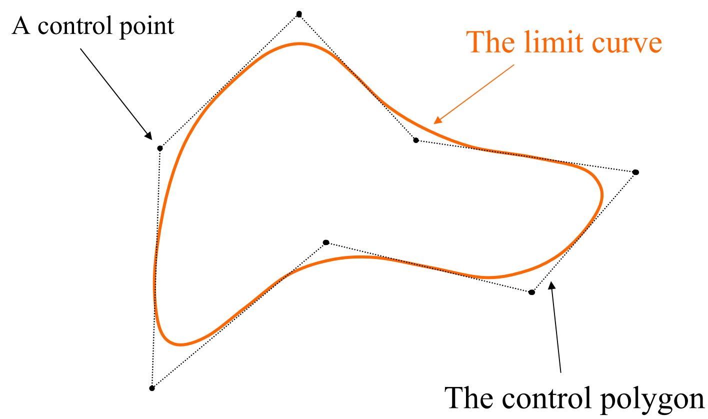
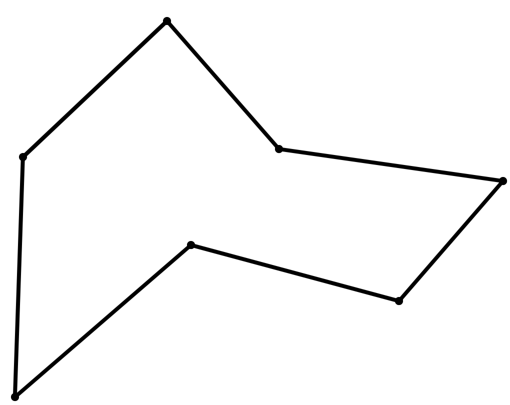


The 4-point scheme



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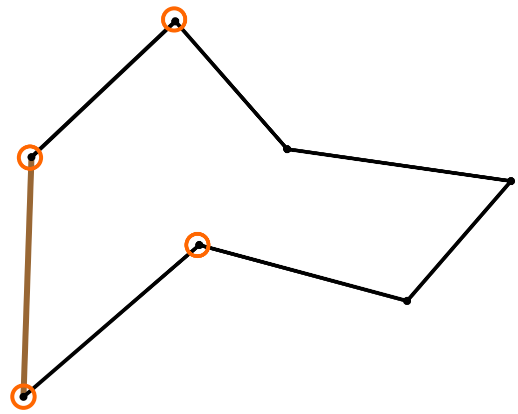

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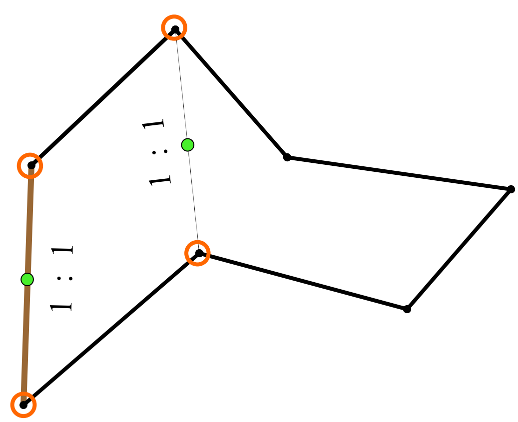

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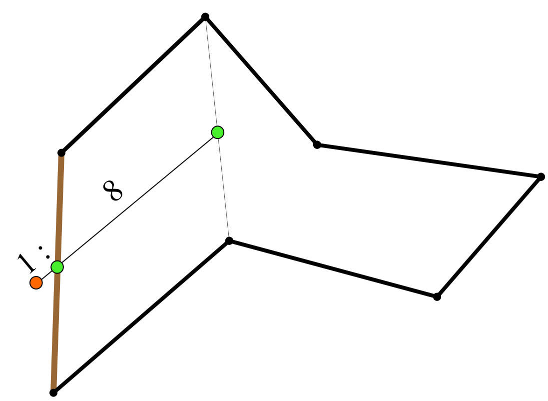

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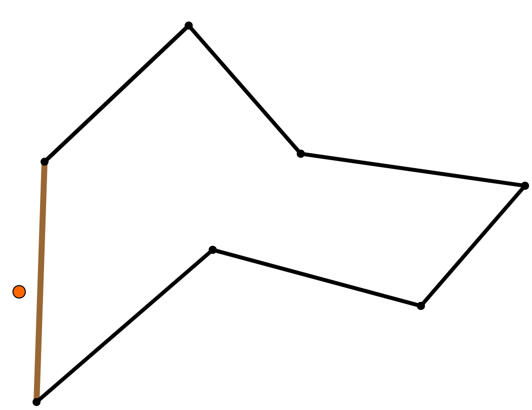

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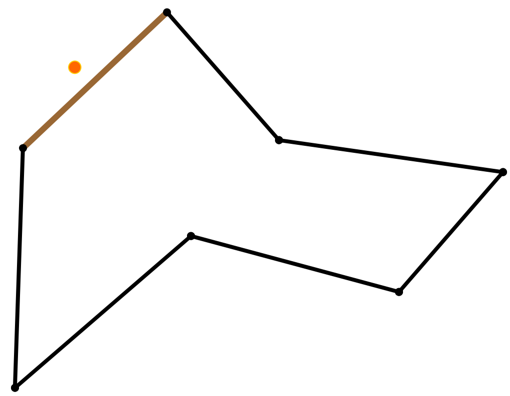

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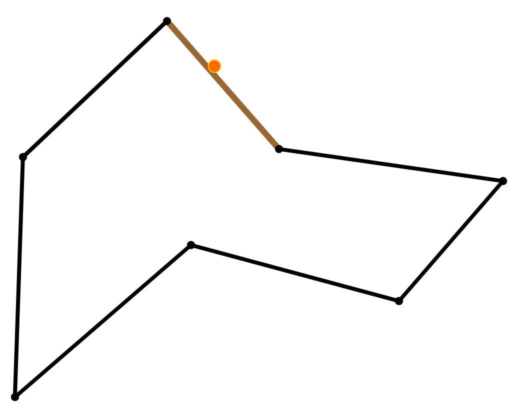

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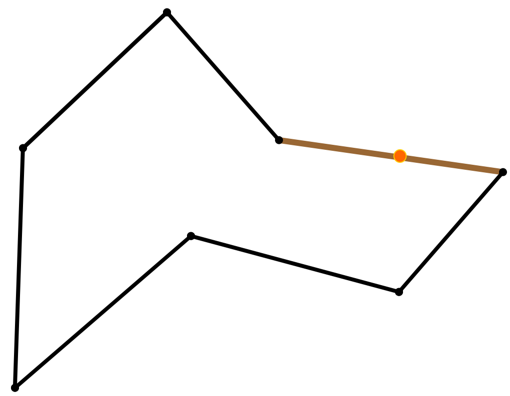

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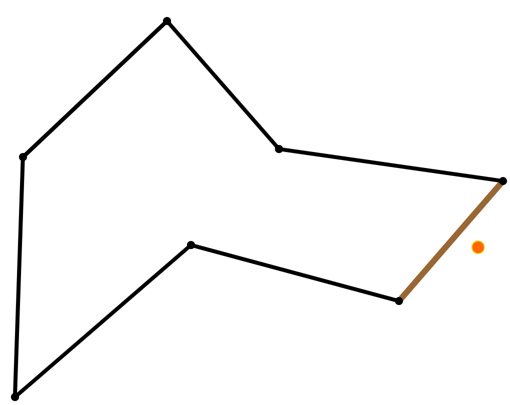

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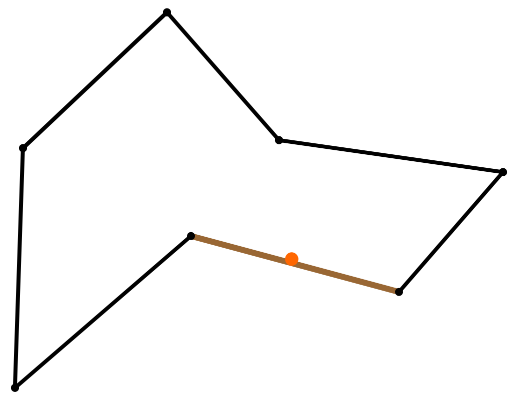

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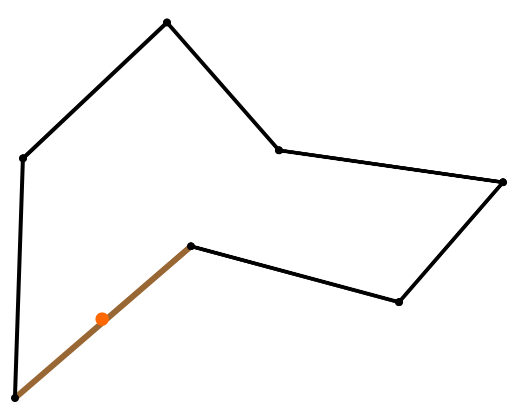

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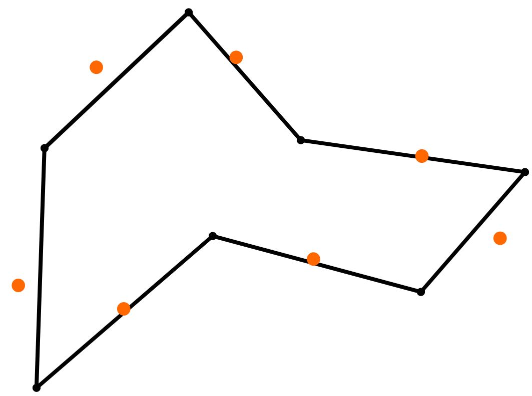

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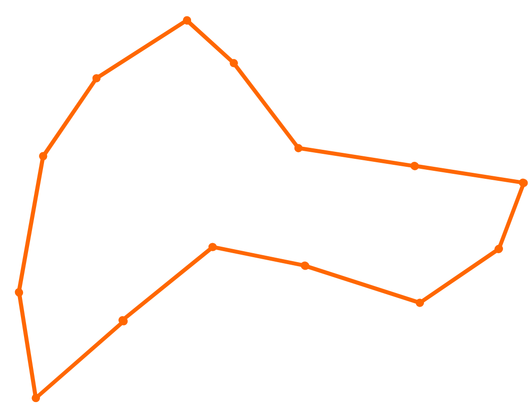

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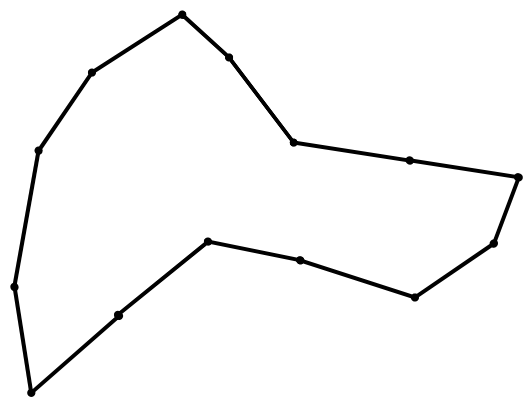

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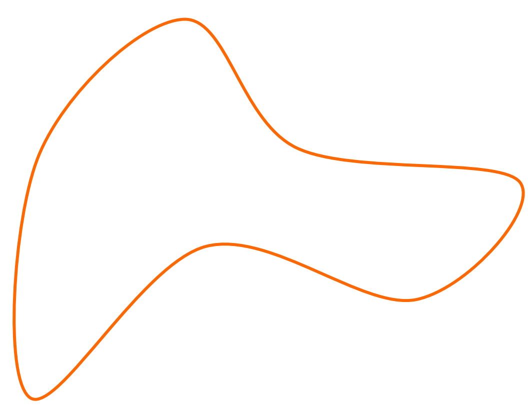

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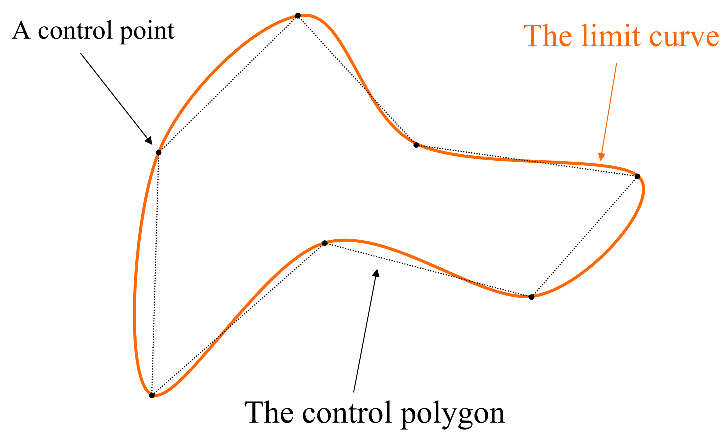
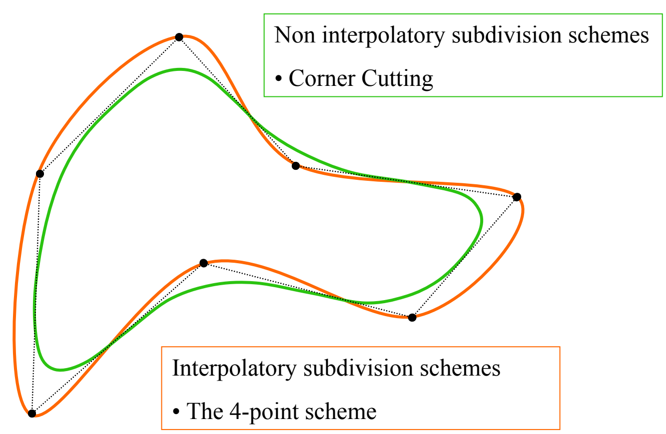


Subdivision curves



Curve Subdivision

- Given a **control polygon**, a **subdivision curve** is generated by repeatedly applying a subdivision operator to it.
- The central theoretical questions:
  - **<u>Convergence</u>**: Given a subdivision operator and a control polygon, does the subdivision process converge?
  - **<u>Smoothness</u>**: Does the subdivision process converge to a smooth curve?

Surface Subdivision

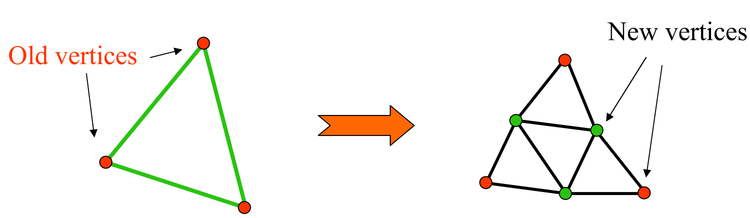
- Given a control net (polygonal mesh consisting of vertices, faces and edges)
- A sudivision surface is geterated by repeatedly
  - Refining the control net – increasing #vertices by factor ~4
  - Applying rules to find position of both new and old vertices

Subdivision Schemes

- In the limit (after ∞ iterations) the control mesh converges to a limit surface
- Usually 2-3 good enough for CG
- Subdivision schemes characterized by
  - Topological refinement rules
  - Rules for calculating position of new vertices

Triangular subdivision

Works only for control nets whose faces are triangular.



Every face is replaced by 4 new triangular faces.

The are two kinds of new vertices:

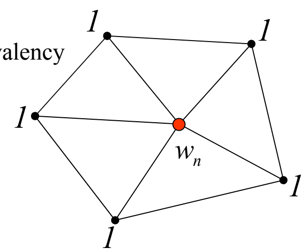
- Green vertices are associated with old edges
- Red vertices are associated with old vertices.

Loop’s scheme

Every new vertex is a weighted average of the old vertices. The list of weights is called the subdivision mask or the stencil.

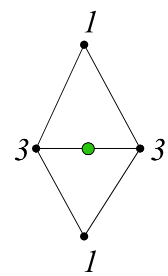
A rule for new red vertices

n - the vertex valency

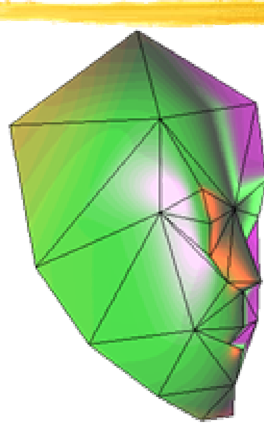


$$w_n = \frac{64n}{40 - \left(3 + 2\cos(2\pi/n)\right)^2} - n$$

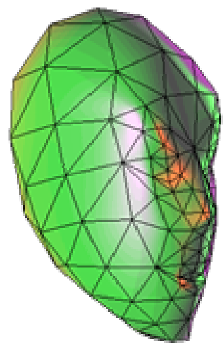
A rule for new green vertices



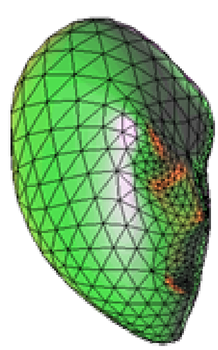
The original control net



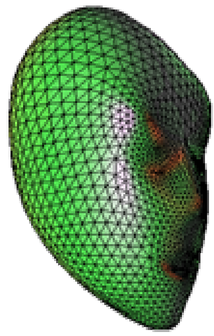
After 1st iteration



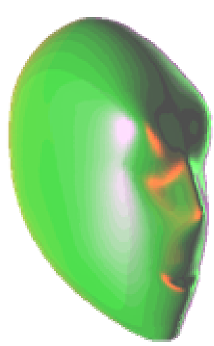
After 2nd iteration



After 3rd iteration



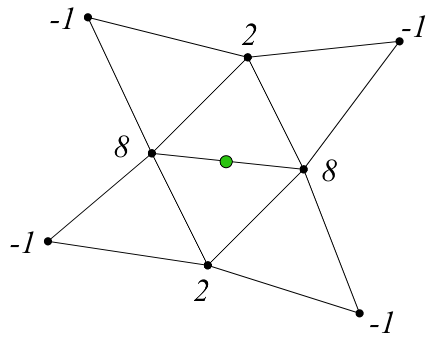
The limit surface



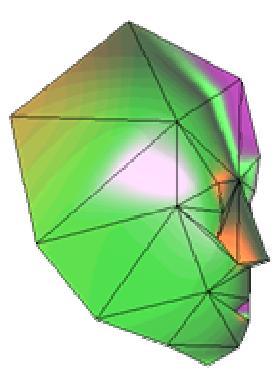
The limit surfaces of Loop's subdivision have continuous curvature almost everywhere.

The Butterfly scheme

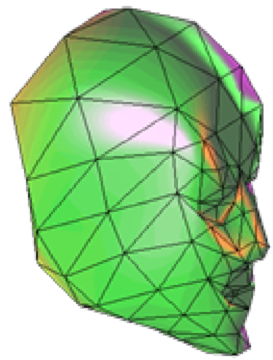
This is an interpolatory scheme. The new red vertices inherit the location of the old vertices. The new green vertices are calculated by the following stencil:



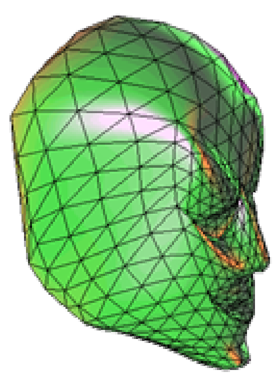
The original control net



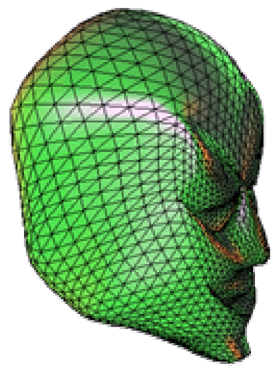
After 1st iteration



After 2nd iteration



After 3rd iteration

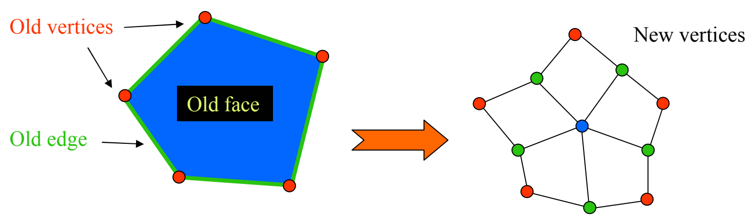


The limit surface

The limit surfaces of the Butterfly subdivision are smooth but are nowhere twice differentiable.

Quadrilateral subdivision

Works for control nets of arbitrary topology. After one iteration, all the faces are quadrilateral.



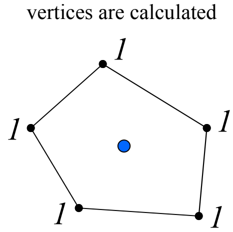
Every face is replaced by quadrilateral faces.
The are three kinds of new vertices:

- Blue vertices are associated with old faces
- Green vertices are associated with old edges
- Red vertices are associated with old vertices.

Catmull Clark's scheme

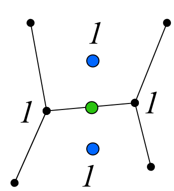
Step 1

First, all the yellow vertices are calculated



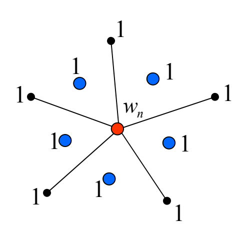
Step 2

Then the green vertices are calculated using the values of the yellow vertices



Step 3

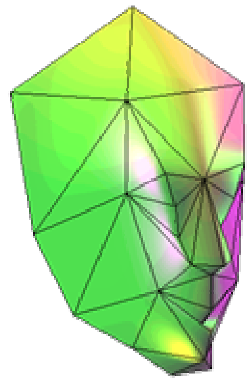
Finally, the red vertices are calculated using the values of the yellow vertices



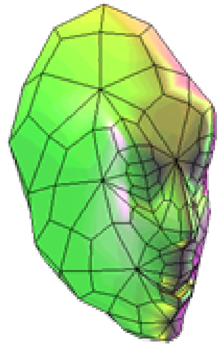
n - the vertex valency

$$w_n = n(n-2)$$

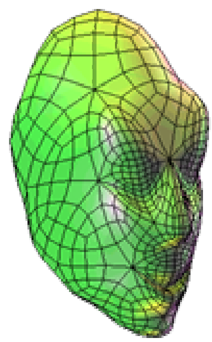
The original control net



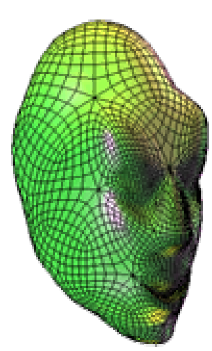
After 1st iteration



After 2nd iteration



After 3rd iteration



The limit surface

The limit surfaces of Catmull-Clarks's subdivision
have continuous curvature almost everywhere.