Radiosity

Radiosity

- Motivation: what is missing in ray-traced images?
  - Indirect illumination effects
  - Color bleeding
  - Soft shadows
- Radiosity is a physically-based illumination algorithm capable of simulating the above phenomena in a scene made of ideal diffuse surfaces.
- Books:
  - Cohen and Wallace, Radiosity and Realistic Image Synthesis, Academic Press Professional 1993.
  - Sillion and Puech, Radiosity and Global Illumination, Morgan-Kaufmann, 1994.

Radiosity in a Nutshell

- Break surfaces into many small elements
- Formulate and solve a linear system of equations that models the equilibrium of inter-reflected light in a scene.
- The solution gives us the amount of light leaving each point on each surface in the scene.
- Once solution is computed, the shaded elements can be quickly rendered from any viewpoint.

Radiometric quantities

- Radiant energy [J]
- Radiant power (flux): radiant energy per second [W]
- Irradiance (flux density): incident radiant power per unit area [W/m^2]
- Radiosity (flux density): outgoing radiant power per unit area [W/m^2]
- Radiance (angular flux density): radiant power per unit projected area per unit solid angle [$W/(m^2\ sr)$]

Solid Angles (1)

- When defining various radiometric quantities, we need to be able to quantify sets of directions.
- In 2D:
  - directions are points on the unit circle;
  - a simply connected set of directions is an arc, whose size corresponds to an angle [radians];
- In 3D:
  - directions are points on the unit sphere;
  - a simply connected set of directions is an area on the sphere, whose size corresponds to a solid angle;
  - solid angles are measured in steradians.

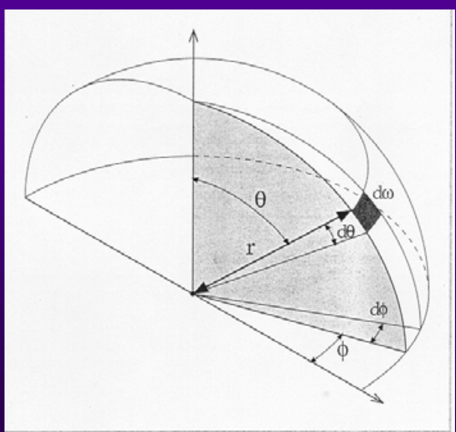
Solid Angles (2)

- Consider the spherical coordinates (r,θ,ϕ) of a point on the sphere. What is the area of a differential surface element at this point?

$$dA = (r\,d\theta)(r\sin\theta\,d\phi) = r^2\sin\theta\,d\theta\,d\phi$$

- Differential solid angle:

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$



The Radiosity Equation (1)

- Assume that surfaces in the scene have been discretized into *n* small elements.
- Assume that each element emits/reflects light uniformly across its surface.
- Define the *radiosity* *B* as the total hemispherical flux density (W/m^2) leaving a surface.
- Let's write down an expression describing the total flux (light power) leaving element *i* in the scene:

total flux = emitted flux + reflected flux

The Radiosity Equation (2)

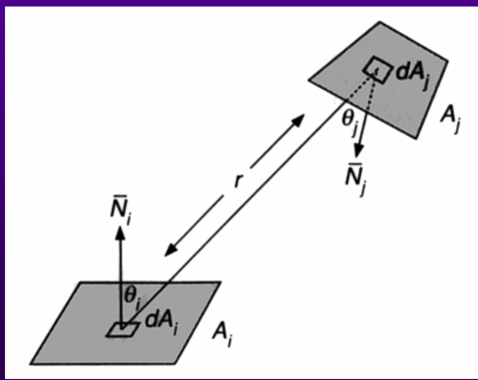
- Total flux leaving element i: $B_i A_i$
- Total flux emitted by element i: $E_i A_i$
- Total reflected flux:
  - (reflectance of element i)*(the total incoming flux)
  - total incoming flux = sum of contributions from all other elements in the scene $\rho_i \sum_j B_j A_j F_{ji}$
- The full radiosity equation is then:

$$B_i A_i = E_i A_i + \rho_i \sum_{j=1}^{n} B_j A_j F_{ji}$$

The Form Factor

- The form factor F_{ij} tells us how much of the flux leaving element i actually reaches element j.

$$F_{ij} = \frac{1}{A_i} \int_{x \in A_i} \int_{y \in A_j} \frac{\cos\theta_x \cos\theta_y}{\pi \|x - y\|^2} V(x,y)\, dA_x\, dA_y$$



Properties of Form Factors

- Reciprocity: $A_i F_{ij} = A_j F_{ji}$
- Additivity: $F_{i(j \cup k)} = F_{ij} + F_{ik}$
- Conservation of energy in a closed environment:

$$\sum_{j=1}^{n} F_{ij} = 1$$

The Radiosity Equation (3)

- The radiosity equation $$B_i A_i = E_i A_i + \rho_i \sum_{j=1}^{n} B_j A_j F_{ji}$$
- Divide equation by A_i: $$B_i = E_i + \rho_i \sum_{j=1}^{n} B_j \tfrac{A_j}{A_i} F_{ji}$$
- Apply form-factor reciprocity: $$B_i = E_i + \rho_i \sum_{j=1}^{n} B_j F_{ij}$$
- We can write this using matrix notation:

$$\begin{bmatrix} B_1 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ \vdots \\ E_n \end{bmatrix} + \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix} \begin{bmatrix} B_1 \\ \vdots \\ B_n \end{bmatrix}$$

Finally...

- A linear system of n equations in n unknowns:

$$\begin{bmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ -\rho_n F_{n1} & \cdots & \cdots & 1-\rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

The Radiosity Method

- Take as input a geometric model of the scene, with emission and reflection properties of each surface
- Step 1 - Meshing: Discretize input surfaces into a mesh of small elements
- Step 2 - Setup: Compute the form factors F_{ij}
- Step 3 - Solution: Solve the resulting linear system of equations
- Step 4 - Display: Render shaded elements from any desired view point.
- These steps are often interleaved in practice.

Examples:

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Solving the Equation

- The naive approach - Gaussian elimination
  - Requires $O(n^2)$ memory to store the matrix
  - Requires $O(n^3)$ time to solve the equation
- A better approach - iterative solution
  - Jacobi iteration
  - Gauss-Seidel iteration
  - Southwell relaxation (known as Progressive Radiosity)
- Due to special properties of the radiosity matrix, it is possible to prove that these iterative methods are guaranteed to converge to the correct solution.

Gauss-Seidel Iteration

- Start with an initial guess: for all i $B_i^{(0)} = E_i$
- Repeatedly compute the radiosities according to the following formula:

$$B_i^{(k)} = \frac{1}{M_{ii}} \left(E_i - \sum_{j=1}^{i-1} M_{ij} B_j^{(k)} - \sum_{j=i+1}^{N} M_{ij} B_j^{(k-1)} \right)$$

- The physical analogy: Each element is "gathering", in turn, light from all other elements.
- This iteration is guaranteed to converge from any initial guess, because the matrix is strictly diagonally dominant.

Progressive Radiosity (1)

- While not converged:
  - Select one element in the scene as the current light source
  - "Shoot" radiosity from the light source to the rest of the scene
- The solution process mimics the physical process of light propagation in the scene.
- Must take care not to shoot the same light more than once (keep track of "unshot radiosity")

```
B[i] = Unshot[i] = E[i]
while (not converged) {
  Choose i with largest Unshot[i]*A[i]
  Shoot(i)
}
```

Progressive Radiosity (2)

```
B[i] = Unshot[i] = E[i]
while (not converged) {
  Choose i with largest Unshot[i]*A[i]
  Shoot(i)
}
```

```
Shoot(i):
  for j = 1..n {
    Compute the form factor FF[i,j]
    Delta[j] = ρ[j] FF[i,j] Unshot[i] A[i]/A[j]
    B[j] += Delta[j]
    Unshot[j] += Delta[j]
  }
  Unshot[i] = 0
```

Progressive Radiosity (3)

- In each iteration the algorithm computes n form factors on the fly, removing the $O(n^2)$ storage complexity.
- Choosing the "brightest" shooter at each iteration makes the solution to converge rapidly during the first iterations.
- It is possible to display the solution after each iteration, resulting in a progressive sequence of images.
- Typically, there is no need to run until complete convergence. The process can be stopped after relatively few iterations.

Progressive Radiosity Example: 0, 8, 16, 25, 50, 100 iterations

The Ambient Correction

- In order to avoid too dark an image during the first PR iterations, an ambient correction term is used.
- Compute the average reflectivity in the scene:

$$\rho_{ave} = \sum \rho_i A_i \Big/ \sum A_i$$

- Define the "overall reflection factor" as:

$$R = 1 + \rho_{ave} + \rho_{ave}^2 + \rho_{ave}^3 + \ldots = \frac{1}{1 - \rho_{ave}}$$

- At each iteration the ambient term is set to:

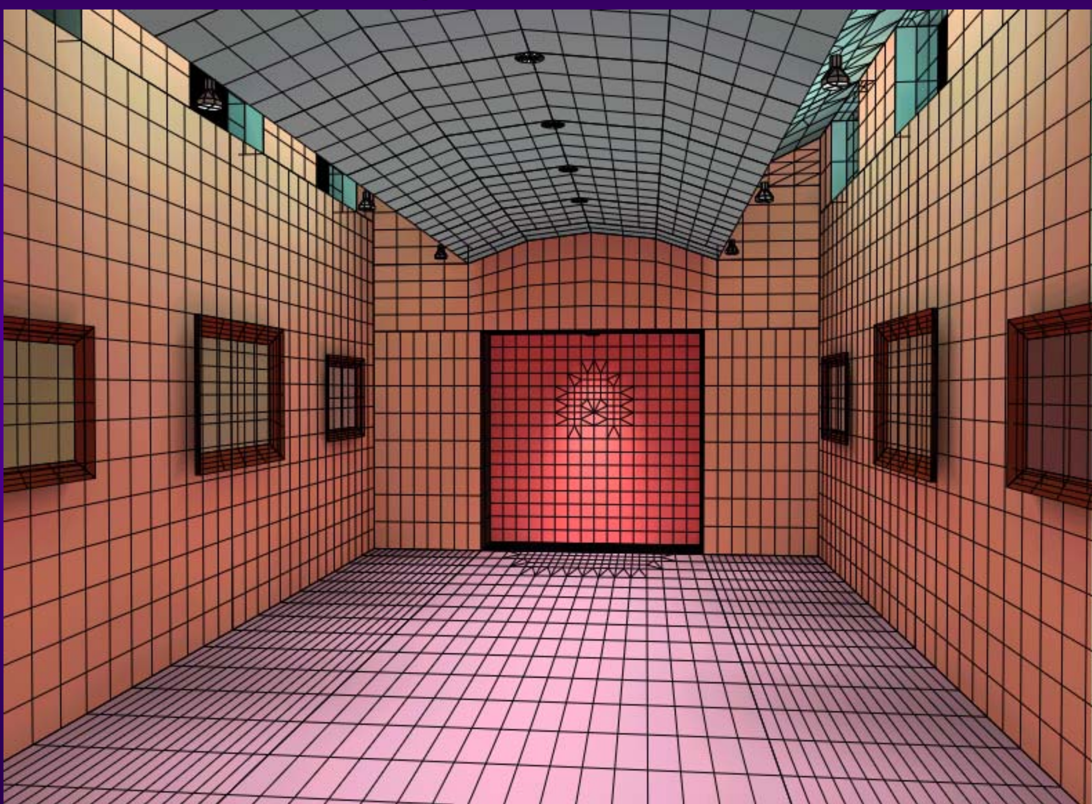
$$Ambient = R\left(\sum Unshot_i A_i\right) \Big/ \sum A_i$$

Progressive Radiosity + Ambient 0, 8, 16, 25, 50, 100 iterations

Adaptive Mesh Refinement

- Start with a coarse mesh of elements.
- During the solution process, subdivide regions in which the radiosity gradient is above a certain threshold.
- Advantage: creates more elements only where necessary.
- Disadvantage: might miss important features altogether.

Progressive Radiosity with Adaptive Meshing (0,8,16,50,100)



Form-Factor Computation (1)

- Analytic as well as approximate formulas exist for various configurations.
- A closed form expression for a form factor between two polygons (Schroeder 93):
  - extremely complicated formula
  - does not take into account occlusion
- A common approximation is to assume that the inner integral is constant for all locations x in element i:

$$F_{ij} = \frac{1}{A_i} \int_{x \in A_i} \int_{y \in A_j} \frac{\cos\theta_x \cos\theta_y}{\pi r^2} V(x,y)\, dA_x\, dA_y$$

$$\approx \int_{y \in A_j} \frac{\cos\theta_x \cos\theta_y}{\pi r^2} V(x,y)\, dA_y$$

Form-Factor Computation (2)

- The remaining integral is approximated as a finite sum:

$$F_{ij} \approx \int_{y \in A_j} \frac{\cos\theta_x \cos\theta_y}{\pi r^2} V(x,y)\, dA_y$$

$$\approx \sum \frac{\cos\theta_x \cos\theta_y}{\pi r^2} V(x,y)\, \Delta A_y$$

- This approximation works well so long as the elements i and j are *well-separated* from each other (the distance between them is large relative to their sizes)

Nusselt's Analogue

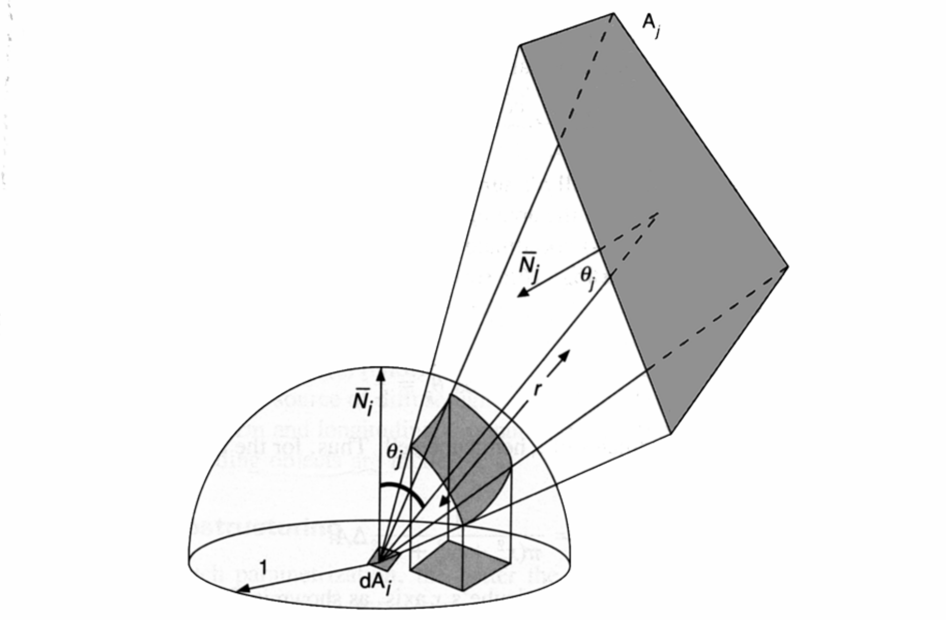


Fig. 16.66 Determining the form factor between a differential area and a patch using Nusselt's method. The ratio of the area projected onto the hemisphere's base to the area of the entire base is the form factor. (After [SIEG81].)

The Hemi-Cube Algorithm
(Cohen - Greenberg 85)

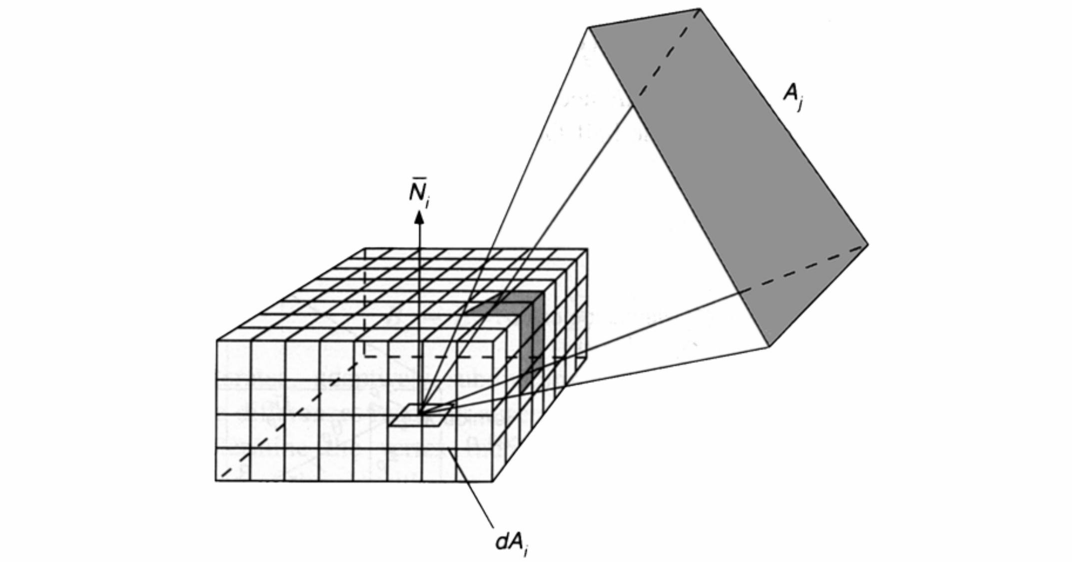


Fig. 16.67 The hemicube is the upper half of a cube centered about the patch. (After [COHE85].)