

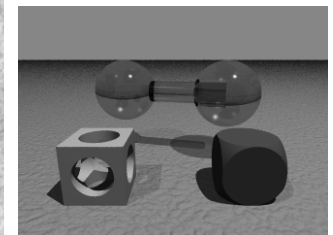
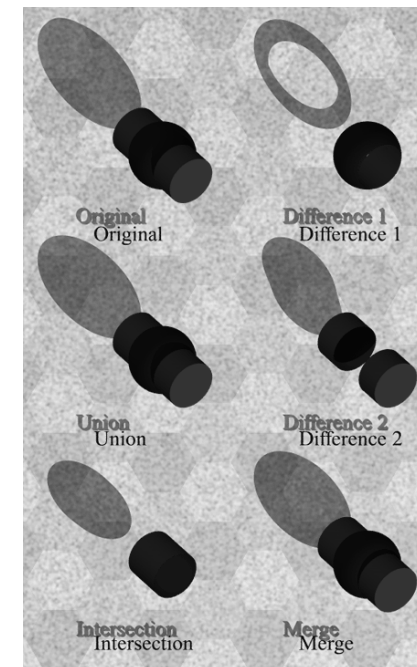
CSG: Constructive Solid Geometry

Solid Geometry

- ◆ So far, we have represented 3D objects by specifying their boundary: this is called Boundary Representation, or B-Rep.
- ◆ Sometimes we need to represent objects explicitly as solids.
- ◆ CSG represents solid objects by constructing them from solid primitives using boolean operations.

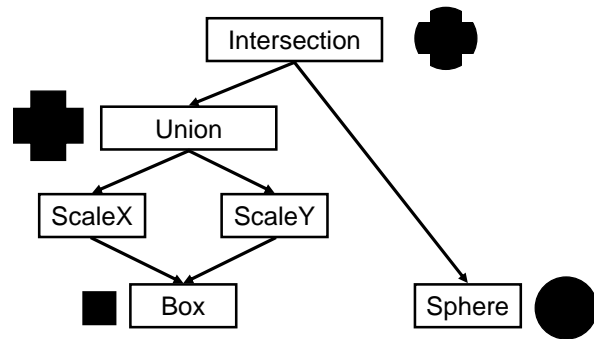
CSG

- ◆ Primitives:
 - ◆ Plane (half-space)
 - ◆ Boxes, spheres, cones, cylinders, ...
- ◆ Affine transformations:
 - ◆ To modify canonical primitives
 - ◆ To get the primitives where we want them
- ◆ Boolean operations:
 - ◆ Union (\cup), Intersection (\cap), Difference (\setminus)



CSG Graph

- ◆ The primitives, affine transformations, and boolean operators are arranged as a DAG: Directed Acyclic Graph:



Point Membership Classification

- ◆ PMC query: given a solid S and a point p , does p belong to S ?
- ◆ PMC(CSGnode A, Point p)

```

If PrimitiveNode(A) return A.DirectPMC(p)
If AffineNode(A) {
  T = A.GetMatrix();
  Return PMC(A.GetChild(), T-1(p))
}
Left = PMC(A.GetLeftChild(), p)
Right = PMC(A.GetLeftChild(), p)
If Intersection(A) return IntersectionTable(Left, Right)
If Union(A) return UnionTable(Left, Right)
If Difference(A) return DifferenceTable(Left, Right)
  
```

Example: Intersection Table

| | | | |
|-----|-----|-----|-----|
| | In | Out | On |
| In | In | Out | On |
| Out | Out | Out | Out |
| On | On | Out | ? |

Regular Sets

- ◆ "Real" solids correspond to regular sets:
 $A \equiv \text{closure}(\text{interior}(A))$
- ◆ To ensure that the result of a boolean operation on two solids is a solid, we "regularize" the result:

$$A \text{ op}^* B = \text{closure}(\text{interior}(A \text{ op} B))$$

for $\text{op} \in \{\cup, \cap, \setminus\}$