Allocation Of Indivisible Goods: A General Model And Some Complexity Results

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ABSTRACT
Many industrial or research activities are so expensive that it is often beneficial for the involved agents to cofund the construction or the purchase of a common required resource. This resource will then be exploited in common, therefore in a shared way. The rules for resource sharing should take account of the possibly antagonistic preferences: each agent wants to maximize its own satisfaction, whereas, from the collective point of view, decisions which both are equitable and exploit the resource in an optimal way are looked for. We give in this article a formal model for indivisible goods resource sharing without monetary compensations and with arbitrary feasibility constraints. We also give some complexity results about this model. The model is applied to a real-world case, namely satellite resource sharing.

Categories and Subject Descriptors
I.2.4 [Computing Methodologies]: Artificial Intelligence—Knowledge Representation Formalisms and Methods

General Terms
Theory

Keywords
fair allocation, resource sharing, preference aggregation, logic, computational complexity.

1. INTRODUCTION
Allocation of indivisible goods has been considered both in social choice theory, where fairness is a key-point but where computational complexity and algorithmics are often neglected, and in artificial intelligence, where research has focused on combinatorial auctions, where fairness is not relevant (see [7]).

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To summarize, an instance of the allocation problem consists of a tuple $(N, O, C, (f, g))$: $N = \{1, \ldots, n\}$, a finite set of agents, $O$, a finite set of objects, $C$, a finite set of constraints over possible allocations, a pair $(f, g)$ of aggregation functions. The output is a feasible allocation maximizing the collective utility.

4. APPLICATION

The latter model can be applied to a real-world problem, that we describe here (see also [3]). The aim of Earth Observing Satellites (EOS) is to acquire images of specified areas on the Earth. Like other space projects, EOS are often co-exploited by several agents, leading to a sharing problem. Moreover, the exploitation of the satellite must obey a set of physical constraints (e.g. memory and energy management). Due to these constraints, the agents’ demands are often conflicting, and the processing center must choose between the requests to be satisfied. The resulting choice has to be both efficient (the satellite cannot be underexploited) and equitable (each agent wants to get a return on investments proportional to its financial contribution).

Each demand of each agent is associated to a weight, that measures the importance of this demand for the involved agent. The requests can express dependencies between images. The satisfaction of an agent can be measured by the sum of the weights of the allocated images. The general quality of an allocation can be ensured by maximizing the collective utility function, whose role must be to guarantee the overall fairness of the sharing. Note that intercomparability of the individual utilities (and thus their normalization) is required.

This application can be formally stated using the previous model. In this framework, the objects are the images, and the constraints and the agents’ demands can be expressed using the appropriate language.

5. RELATED WORK AND CONCLUSION

Allocation of indivisible goods has widely been considered, both in social choice theory and in artificial intelligence. Most works concerning fair resource allocation in social choice theory deal with divisible goods. There are a few recent works about fair division of indivisible goods; they generally do not consider compact representation issues. For example, the work [4] studies approximation schemes for another fairness criterion: envy-freeness (see also [1]).

On the contrary, several “bidding languages” have been developed in the community of combinatorial auctions (see e.g. [6] for a recent overview). Most of these languages allow for expressing complex preferences about bundles. Complexity issues have been studied, for the winner determination problem (that is known to be NP-hard), and some restrictions that make it tractable. However, fairness is irrelevant in combinatorial auctions, which corresponds to a purely utilitarianistic resource allocation. ¹

6. REFERENCES


¹A long version of this article is available at http://www.cert.fr/dsd/THESIS/abouveret/resources/AAMAS/aig.pdf