A Decentralized Bargaining Protocol on Dependent Continuous Multi-Issue for Approximate Pareto Optimal Outcomes

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ABSTRACT

Negotiation techniques have been demonstrated to be effective in solving complex multi-objective problems. When the optimization process operates on continuous variables, it can be tackled by agents bargaining with different objectives. However, the complexity of highly reconfigurable scenarios with a large number of agents does not allow the adoption of classical game theory techniques to design optimal negotiation protocols [2]. We present a decentralized bargaining protocol on dependent continuous multi-issue that produces approximate Pareto optimal outcomes between two agents.

Categories and Subject Descriptors

I.2.11 [**Distributed Artificial Intelligence**]: Multiagent Systems; G.4 [**Mathematical Software**]: Algorithm Design and Analysis.

General Terms

Algorithms, Economics, Experimentation.

Keywords

Decentralized Bargaining, Nash Bargaining Solution, Pareto Optimality.

1. INTRODUCTION

We describe an approach based on a bargaining protocol to produce, in a decentralized way, Pareto optimal outcomes between two agents [1]. Our proposal is a first step towards a decentralized many-to-many bargaining model satisfying the Nash bargaining solution criterion [3]. Given two agents and a mediator, the agents reach an agreement via a sequence of interleaved offers of the agents to the mediator and counter-offers of the mediator to the agents. We designed the negotiation protocol and the negotiation functions (i.e., the strategies) with which the agents calculate their offers in order to obtain approximate Pareto optimal outcomes.

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Our approach presents some interesting properties: its computational complexity is independent of the number of issues to negotiate, it is decentralized and does not require any entity with complete information about the problem, and it can be employed for any utility functions the agents embed.

In the following, we describe the algorithm for producing approximate Pareto optimal outcomes and we present its preliminary experimental evaluation.

2. THE BARGAINING APPROACH

We consider a decentralized bargaining protocol on dependent continuous multi-issue performed concurrently by two agents and a mediator, in which the two agents bargain individually with the mediator. More precisely, we model this bargaining process with two sub-bargaining processes, each one performed by a single agent i and the mediator. These sub-bargaining processes are simultaneous (they are performed concurrently), dependent (a sub-bargaining affects the other sub-bargaining), and synchronous (the temporal line is shared by the two agents and the mediator). The two sub-bargaining processes are carried on with a sequence of offers performed by the agents and counter-offers performed by the mediator. We call $\mathbf{p}_i^t \in I$ – where $I \subseteq \Re^m$ – the offer of the agent i to the mediator at time t, and $\mathbf{a}^t \in I$ the agreement (counter-offer) of the mediator to the agents at time t. Each sub-bargaining process can be represented (for $i \in [1, 2]$) as: $\mathbf{p}_i^0 \succ \mathbf{a}^0 \succ \mathbf{p}_i^1 \succ \cdots \succ \mathbf{a}^\tau$, where τ is the instant of time at which the agents agree, and, consequently, the bargaining process terminates.

For our purposes, each agent i embeds an utility function $\mathcal{U}_i : I \rightarrow \Re$ and a negotiation function $\mathcal{F}_i$ that gives the proposal $\mathbf{p}_i^{t+1}$ of the agent i at the instant of time $t+1$. The mediator computes its counter-proposal at time t according to a function $\mathcal{A} : I \times I \rightarrow I$ that defines the agreement ($\mathbf{a}^t$) at that time harmonizing the two proposals. The agreement $\mathbf{a}^\tau$ computed at time τ is the outcome of the bargaining.

We want to design $\mathcal{F}_i$ and $\mathcal{A}$ in order to produce Pareto optimal outcomes. We exploit geometrical information about the utility functions in the Pareto optimal outcomes. In the case of two bargainers, given an outcome $\mathbf{o}$, the iso-level curves $\mathcal{U}_1 = \mathcal{U}_1(\mathbf{o})$ and $\mathcal{U}_2 = \mathcal{U}_2(\mathbf{o})$ are tangent in $\mathbf{o}$ if and only if $\mathbf{o}$ is Pareto optimal [3]. However, in a decentralized scenario, agents have not any knowledge about other agent's utility function and each agent can take into account only its own utility function and the counter-offers of the mediator. A solution is that each agent produces a proposal $\mathbf{p}_i^{t+1}$ in the attempt to (α) make its proposal closer and closer to the agreement $\mathbf{a}^t$ and (β) make the difference between its proposal and the agreement ($\mathbf{p}_i^{t+1} - \mathbf{a}^t$) orthogonal to the tangent to the iso-level curves of its utility function. We assume that $\mathcal{A}$ is the arithmetic mean between the proposals of the two agents. In this way the differences between the proposals of the two agents and the agreement (i.e., $(\mathbf{p}_1^{t+1} - \mathbf{a}^t)$ and $(\mathbf{p}_2^{t+1} - \mathbf{a}^t)$) have the same direction. Then, according to (β) the tangents to the iso-level curves of the two agents in their proposals are parallel. Thus, if the proposals of the two agents converge satisfying (β), the outcome is Pareto optimal.

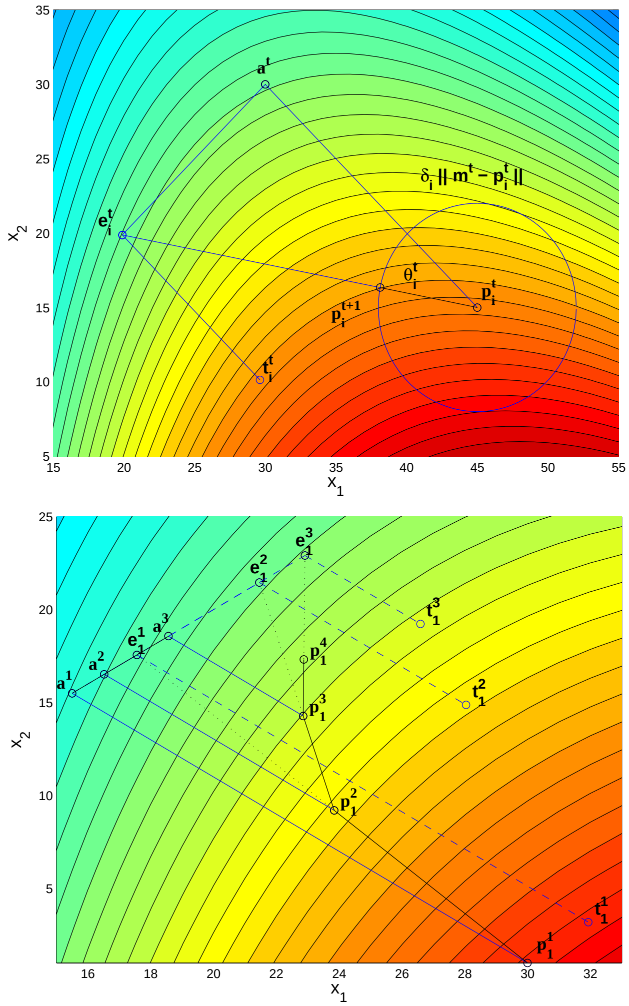

Figure 1: Determination of the proposal $\mathbf{p}_i^{t+1}$ (top) and three periods of the sub-bargaining of agent 1 (bottom) in a 2D space

We present an approximate algorithm to obtain the above result. Formally, the proposal of agent i is produced as follows (Fig. 1 (top)). We define a versor $\mathbf{v}_i^t$:

$$\mathbf{v}_i^t = \frac{\mathbf{a}^t - \mathbf{p}_i^t}{\|\mathbf{a}^t - \mathbf{p}_i^t\|},$$

that is along the direction of the difference between $\mathbf{a}_i^t$ and $\mathbf{p}_i^t$. We call $\mathbf{t}_i^t$ the point belonging to the iso-level curve $\mathcal{U}_i = \mathcal{U}_i\left(\mathbf{p}_i^t\right)$ in which the gradient $\nabla\mathcal{U}_i(\mathbf{t}_i^t)$ is directed as $\mathbf{v}_i^t$. This means that the tangent to such iso-level curve in $\mathbf{t}_i^t$ is orthogonal to $\mathbf{v}_i^t$:

$$\begin{cases} \left| \dfrac{\nabla\mathcal{U}_i(\mathbf{t}_i^t)}{\|\nabla\mathcal{U}_i(\mathbf{t}_i^t)\|} \cdot \mathbf{v}_i^t \right| = 1 \\ \mathcal{U}_i(\mathbf{t}_i^t) = \mathcal{U}_i(\mathbf{p}_i^t) \end{cases}$$

We define $\mathbf{e}_i^t$ as the projection of $\mathbf{t}_i^t$ on the direction orthogonal to $\mathbf{v}_i^t$ that passes in $\mathbf{a}^t$:

$$\mathbf{e}_i^t = \left(\left(\mathbf{a}^t - \mathbf{t}_i^t\right) \cdot \mathbf{v}_i^t\right) \mathbf{v}_i^t + \mathbf{t}_i^t$$

The proposal of the agent is produced in the attempt to get closer to $\mathbf{e}_i^t$ with a step equal to $\delta^t \|\mathbf{a}^t - \mathbf{p}_i^t\|$:

$$\mathbf{p}_i^{t+1} = \mathcal{F}_i\left(\mathbf{p}_i^t, \mathbf{a}^t\right) = \mathbf{p}_i^t + \delta^t \cdot \|\mathbf{a}^t - \mathbf{p}_i^t\| \cdot \frac{\mathbf{e}_i^t - \mathbf{p}_i^t}{\|\mathbf{e}_i^t - \mathbf{p}_i^t\|},$$

where $\delta^t \in [0, 1]$ is an a-dimensional parameter (it does not scale with the application). Since the agreement is the arithmetic mean, $\|\mathbf{a}^t - \mathbf{p}_i^t\|$ is the same for all i. Fig. 1 (bottom) shows the first three periods of a sub-bargaining between agent 1 and mediator.

3. CONCLUSIONS

We evaluated the performances of the algorithm employing utility functions typically adopted in Cournot game [3]. In Fig. 2 we compare, in the space of the normalized utilities, the outcomes obtained with the proposed approach with the Pareto frontier. We note that most of the outcomes lay on the Pareto frontier, except a few outcomes that are located around $(\mathcal{U}_1^*, \mathcal{U}_2^*) = (0.475, 0.475)$. The reason is that the initial proposals of the agents were too close and the bargaining is terminated before reaching an optimal outcome. To overcome this drawback a refinement of the protocol is needed.

We aim to further develop the bargaining protocol presented in this work to improve its performances and to support many-to-many bargaining.

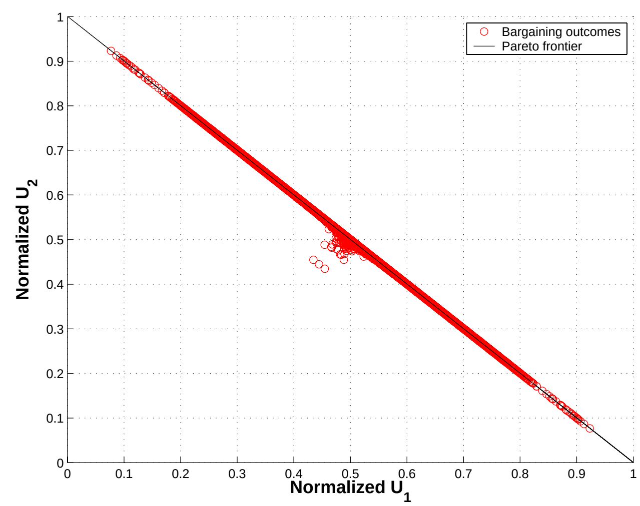

Figure 2: Pareto frontier and bargaining outcomes

4. REFERENCES

[1] P. Heiskanen. Decentralized method for computing pareto solutions in multiparty negotiations. *EUR J OPER RES*, 117:578–590, 1999.

[2] S. Parsons and M. Wooldridge. Game theory and decision theory in multi-agent systems. *AUTON AGENT MULTI-AG*, 5(3):243–254, 2002.

[3] A. E. Roth. *Game-Theoretic Models of Bargaining*. Cambridge University Press, Cambridge, USA, 1985.