Graphical Models - problem set 5
December 15, 2004

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1. Recall that in variational approximate inference we want to minimize the free energy defined by
$$F(q,y) = \sum_x q(x)E(x,y) + \sum_x q(x)ln(q(x))$$
where $E(x,y) = -ln(P(x,y))$.

   (a) Write the *Mean Field* approximation F_{MF} for $F(q,y)$ for a Markov random field where the potentials are defined over pairs of variables x_i and x_j as well as local potentials Ψ_{ii}. Remember that the Mean Field approximation assumes $q(x) = \prod_i q_i(x_i)$

   (b) Show that in order to minimize the above F_{MF} we need to set
$$q_i(x_i) = \alpha \Psi_{ii} \exp\left(\sum_{j \in N(i)} \sum_{x_j} q_j(x_j) log(\Psi_{ij})\right)$$
   Note: Remember to include the constraint $\sum_{x_i} q_i(x_i) = 1$ for all i in your derivations to ensure proper probabilities.

   (c) Show that if, when updating $q_i(x_i)$'s, all other q's are fixed then F_{MF} does not increase as a results of the update.

   (d) Use (b) and (c) to suggest a practical Mean Field approximation.

2. In Gibbs sampling we have shown that at each time step we need to sample from $p(X_i|X_1,\ldots,X_{i-1},X_{i+1},\ldots,X_n)$. Show That how can we do so if our model is a

   (a) Bayesian network

   (b) Markov network