Graphical Models - problem set 3
November 17, 2004

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1. Recall the definitions

$$\Phi_{ij\downarrow} = \prod_{k,l \in ij\downarrow} \Psi_{kl}(x_k, x_l) \prod_{k \in ij\downarrow} \Psi_{kk}(x_k, y_k) \tag{1}$$

   and

$$m_{ji}(x_i) = \sum_{x_{ij\downarrow}} \Psi_{ij}(x_i, x_j)\Phi_{ij\downarrow}(x_{ij\downarrow}) \tag{2}$$

   That is, $m_{ji}(x_i)$ is the product of all cliques from the side of the tree that is separated from i by j times the clique over i and j (this is **all** of the "information" that i needs to get from j's side).

   Use the above definitions directly (and NOT the recursion formula for m_{ij}) to show that having calculated all messages m_{ij} in the graph, we can exactly calculate $P(x_i, x_j|y)$ for all connected pairs of nodes in the graph. (hint: try to generalize the equation that we derived for HMMs).

2. Recall that $P(x_i|y)$ can be written as a product of incoming "messages":

$$P(x_i|y) = c\Psi_{ii}(x_i, y_i) \prod_{j \in N(i)} m_{ji}(x_i) \tag{3}$$

   Use this property, the results of the previous question and the marginalization constraint:

$$P(x_i|y) = \sum_{x_j} P(x_i, x_j|y) \tag{4}$$

   to give an alternative proof for the recursion over messages:

$$m_{ji}(x_i) = \sum_{x_j} \Psi_{ij}(x_i, x_j)\Psi_{jj}(x_j, y_j) \prod_{k \in N(j)\setminus i} m_{kj}(x_j) \tag{5}$$