Graphical Models - problem set 2

Submission date: Wednesday, 17/11

1. Consider the "Asia" domain where the chances for "Dyspnea" (a rare lung disease) are evaluated. The variables in the domain are [V]isit to Asia, [T]uberculosis, [S]moking, [L]ung cancer, [B]ronchitis, [A]bnormality in chest, [X]-ray and of course [D]yspnea. Now assume that the distribution factorizes as

$$\begin{aligned} P(V,T,S,L,B,A,X,D) &= \alpha\Psi_{VT}(vt)\cdot\Psi_{TL}(tl)\cdot\Psi_{SL}(sl)\cdot\Psi_{SB}(sb)\cdot\Psi_{LA}(la) \\ &\cdot\Psi_{TA}(ta)\cdot\Psi_{AB}(ab)\cdot\Psi_{AD}(ad)\cdot\Psi_{BD}\cdot\Psi_{AX}(ax) \end{aligned}$$

   for the undirected model and as

$$\begin{aligned} P(V,T,S,L,B,A,X,D) &= P(V)\cdot P(T|V)\cdot P(S)\cdot P(L|S)\cdot P(B|S) \\ &\cdot P(A|T,L)\cdot P(D|A,B)\cdot P(X|A) \end{aligned}$$

   for the directed model.

   Draw the corresponding (directed and undirected) graphs such the distribution are Markovian with respect to the graph. Using these graphs (no need for formal proof) answer the following independenceis queries for both models:

   (a) $(T \perp L)$?
   (b) $(T \perp L \mid A)$?
   (c) $(S \perp X \mid L)$?
   (d) $(S \perp D \mid A, B)$?
   (e) $(V \perp L \mid T, A)$?

2. Assuming all variables are binary, 0 denoting "No" and 1 denoting "Yes", here are some of the potentials for the above example (for each potential, the rows match assignment for the first variable and the columns for the second)

$$\Psi_{VT} = \begin{pmatrix} 80 & 1 \\ 70 & 15 \end{pmatrix} \quad \Psi_{TL} = \begin{pmatrix} 1.2 & 1 \\ 1 & 1.2 \end{pmatrix} \Psi_{LA} = \begin{pmatrix} 5 & 40 \\ 6 & 85 \end{pmatrix} \Psi_{TA} = \begin{pmatrix} 3 & 70 \\ 3 & 105 \end{pmatrix}$$

   (a) Explain the logic of any of the above potentials and use similar logic to define any other reasonable potentials needed in order to answer the query $P(T|V, L, S, B, A, D, X)$.
   (b) Calculate the probability of Tuberculosis given a Visit to Asia of a non-smoker with lung cancer, no Bronchitis, with abnormality in chest, no Dyspnea and a negative X-ray.

3. In class we saw that if $\boldsymbol{X}$ is globally Markov with respect to a graph $\boldsymbol{G}$ then it both locally Markov with respect to a graph $\boldsymbol{G}$ and pairwise Markov with respect to $\boldsymbol{G}$. Give an example of a distribution $\boldsymbol{X}$ and a graph $\boldsymbol{G}$ that is locally Markov or pairwise Markov, but is not globally Markov.

   Hint 1: Think of several deterministic relations that ensure certainty that is not ensured when considering variables that are further apart.

   Hint 2: Use at least one deterministic relation that involves more then 2 variables.