Graphical Models - problem set 1

October 31, 2004

Due date: Sunday, 7/11

1. In class we proved that $\alpha_t(i)$ satisfies a recursion formula. Show that $\beta_t(j)$ satisfies the backward recursion:

$$\beta_t(j) = \sum_{i=1}^{N} A(j,i)B(i,y_{t+1})\beta_{t+1}(i)$$

2. Assume that you have calculated $\alpha_t(i)$ and $\beta_t(j)$ for all values of t,i,j. Show how to use this to compute

$$P(X_{t+1} = j \mid X_t = i, Y)$$

for any t,i and j.

3. Given a point $\boldsymbol{x}^0$ in a d dimensional space ($\boldsymbol{x} = (x_1..x_d)$) find the closest point to it on the hyper plane $\sum_{i=1}^{d}(x_i) = 0$. In other words find :

$$\arg\min_{\boldsymbol{x}} \frac{1}{2} \left\| \boldsymbol{x} - \boldsymbol{x}^0 \right\|^2 \tag{1}$$

$$\text{such that } \sum_{i=1}^{d}(x_i) = 0$$

where $\|\boldsymbol{x} - \boldsymbol{x}^0\| = \sqrt{\sum_{i=1}^{d}(x_i - x_i^0)^2}$