

## Formulas and definitions for the DAST exam

Thursday, June 24, 2004.

### Master Theorem

Let  $a \geq 1, b > 1$  be constants, let  $f(n)$  be a function, and let  $T(n)$  be defined on non-negative integers by the recurrence:  $T(n) = aT(n/b) + f(n)$  where  $n/b$  is either  $\lceil n/b \rceil$  or  $\lfloor n/b \rfloor$ :

1. If  $f(n) = O(n^{\log_b a - \varepsilon})$ ,  $\varepsilon > 0$  then  $T(n) = \Theta(n^{\log_b a})$
2. If  $f(n) = \Theta(n^{\log_b a})$  then  $T(n) = \Theta(n^{\log_b a} \lg n)$
3. If  $f(n) = \Omega(n^{\log_b a + \varepsilon})$ ,  $\varepsilon > 0$  and if  $a f(n/b) \leq c f(n)$  for some  $c < 1$  and a sufficiently large  $n$ , then  $T(n) = \Theta(f(n))$

### Logarithms

$$\log_c(ab) = \log_c a + \log_c b$$

$$\log_b a^n = n \log_b a$$

$$\log_b(1/a) = -\log_b a$$

$$\log_b a = 1/\log_a b$$

$$\lg n = \log_2 n$$

$$a = b^{\log_b a}$$

$$a^{\log_b c} = c^{\log_b a}$$

### Arithmetic series

$$\sum_{i=1}^n k = \frac{1}{2} n(n+1)$$

$$\sum_{i=1}^n k^2 = \frac{1}{6} n(n+1)(2n+1)$$

$$\sum_{i=1}^n k^3 = \frac{1}{4} n^2(n+1)^2$$

### Geometric series

$$\sum_{i=0}^k c^i = \frac{c^{k+1} - 1}{c - 1} \quad \text{for } |c| > 1$$

$$\sum_{i=0}^{\infty} c^i = \frac{1}{1-c} \quad \text{for } |c| < 1$$

### Harmonic series

$$\sum_{i=1}^n \frac{1}{k} = \ln n + O(1)$$