# Human Motion Tracking and Orientation Estimation using inertial sensors and RSSI measurements 

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science
by
Ami Luttwak

Supervised by
Prof. Danny Dolev

School of Engineering and Computer Science The Hebrew University of Jerusalem

Israel

December 28, 2011

## Acknowledgments

I wish to express my deepest gratitude to Prof. Danny Dolev. For his excellent guidance, encouragement and great patience throughout this research. This thesis would not have been possible without the devotion, cooperation and an endless will to help of both Dr. Gaddi Blumrozen and Bracha Hod. I am sincerely grateful for their help, guidance and academic cooperation. I would also like to thank to Dr. Tal Anker for his continuous support. Finally, I wish to thank my parents for their unmeasurable support and love.

## Abstract

Real-time tracking of human body motion is an important technology in biomedical applications, robotics, and other human computer interaction applications. Inertial sensors can overcome problems with jitter, latency interference, line-of-sight obscurations and limited range but suffer from slow drift. RSSI based location estimation can detect location without drifting but is slow, sensitive to shadowing effects and suffers from large estimation errors. For these reasons, most research considered RSSI-based estimation unsuitable for Body Area Network environment. Recent work in our lab, demonstrated an RSSI-based setup able to perform well in close proximity, rendering it useful for human motion tracking. This paper describes the design of a complementary kalman filter to integrate the data from these two types of sensors in order to achieve the excellent dynamic response of an inertial system without drift and without the shadowing sensitivity of RSSI-based estimations. Real-time implementation and testing results of the complementary Kalman filter are presented. Experimental results validate the filter design, and show the feasibility of using inertial/RSSI sensor modules for real-time human body motion tracking.

## Table of Contents

Acknowledgments ..... 3
Abstract ..... 4
1 Introduction ..... 14
1.1 Motivation ..... 15
1.2 Thesis Contribution ..... 15
1.3 Thesis Outline ..... 16
2 Related Work ..... 17
2.1 Inertial motion tracking ..... 17
2.2 Reducing drift in inertial systems ..... 19
2.2.1 sensor fusion ..... 19
2.2.2 Domain specific assumptions ..... 21
2.3 RSSI-based tracking ..... 22
3 System Model ..... 24
3.1 System description ..... 24
3.2 Inertial sensors modeling ..... 24
3.2.1 Rotations and Sensor co-ordinate system ..... 25
3.2.2 The Gyroscope sensor model ..... 27
3.2.3 The Accelerometer sensor model ..... 28
3.3 RSSI modeling ..... 28
4 Location estimation ..... 31
4.1 Location estimation using inertial sensors ..... 31
4.1.1 Strapdown inertial navigation ..... 32
4.1.2 Domain-specific assumptions - the orientation kalman filter ..... 36
4.2 Location estimation using RSSI ..... 42
4.3 Location estimation using sensor fusion ..... 44
5 Experimental methods ..... 55
5.1 Experiment setup ..... 55
5.1.1 Inertial system setup ..... 56
5.1.2 RSSI system setup ..... 57
5.2 Comparison with optical tracking system ..... 59
5.3 Model parameters estimation ..... 59
5.4 Sensor calibration ..... 60
5.5 Data Synchronization ..... 64
5.6 Analysis ..... 67
6 Results ..... 68
6.1 Parameter identification ..... 68
6.2 Typical results ..... 68
6.3 Gyroscope bias estimation ..... 71
6.4 Accelerometer bias estimation ..... 74
6.5 Sensitivity to initial condition errors ..... 74
7 Discussion and Future Work ..... 79
8 Conclusions ..... 82
Bibliography ..... 83

## List of Figures

2.1 Comparison of position drift performance of commercial, tactical, navigation, strategic-grade, and perfect pure inertial navigation systems [1]. Note that commercial and tactical grade systems are rendered useless after just a few seconds. Also, even the "perfect" ins eventually drifts due to fluctuations in earth's gravitational field.
3.1 System description diagram. The diagram details the system's components, the mobile node, the anchor nodes and the control station and the relationship between them. The mobile nodes periodically sends a data packet to the anchor nodes, which compute the Recieved Signal Stregnth. The control station aggregates, synchronizes and estimates location utilizing both inertial and RSS-based estimations
3.2 Sensor model diagram. The diagram details the components of each signal $\left(\mathbf{y}_{G, t}\right.$ and $\left.\mathbf{y}_{A, t}\right)$ and the relationships between them. Gyroscope signal is modeled as the angular velocity component $\omega_{t}$, a white noise component $\mathbf{v}_{G}$ and a slowly varying gyroscope bias $\mathbf{w}_{b g}$. The orientation ${ }^{G S} R$ is computed from the gyroscope signal using the box 'integration'. The acceleration signal $\mathbf{y}_{A, t}$ is composed of the acceleration and gravity vector relative to the sensor's frame $\left({ }^{S} a-{ }^{S} g\right)$, a white measurement noise $\mathbf{v}_{A}$
4.1 Top level design of the inertial sensors based estimation system. The two inertial sensor systems, 3D accelerometer and 3D gyroscope send their data to the INS component $\left(\mathbf{y}_{A, t}\right.$ and $\mathbf{y}_{G, t}$ respectively). Integrating the data, the INS main component estimates the location, velocity and orientation ( $\hat{\mathbf{p}}_{\text {in }}, \hat{\mathbf{v}}_{\text {in }}$ and $\hat{\theta}_{\text {in }}$ respectively). An orientation kalman filter is used to improve robustness of the system the face of gyroscope bias. The kalman filter uses orientation and acceleration data ( $\hat{\theta}_{\text {in,t }}$ and $\mathbf{y}_{A, t}$ respectively) to estimate orientation and bias errors $\left(\theta_{\varepsilon}, b_{g, \varepsilon}\right)$. These errors are used in a feedback loop by the INS component to correct future estimations. The kalman filter uses the covariance matrices of the accelerometer white noise, gyroscope white noise and gyroscope bias noise to optimally estimate errors $\left(Q_{v A}, Q_{v G}, Q_{w g}\right)$. . . . . . .
4.2 Strapdown intertial navigation algorithm. The angular velocity $\hat{\omega}_{i n, t}$ is calculated from the gyroscope signal $\mathbf{y}_{G, t}$ minus the estimated bias. Strapdown integration is then used to produce the orientation estimate ${ }^{G S} \hat{\mathbf{R}}_{t}$. Acceleration is calculated from the accelerometer signal $\mathbf{y}_{A, t}$ minus the estimated bias, rotated to the global frame minus the gravity vector. Finally, the velocity ${ }^{G} \hat{v}_{t}$ is found by integrating the acceleration ${ }^{G} \hat{a}_{t}$, and location ${ }^{G} \hat{p}_{t}$ is found by integrating the velocity.
4.3 Structure of the kalman filter. Two estimations of the inclination are used, $\hat{\mathbf{Z}}_{G}^{-}$and $\hat{\mathbf{Z}}_{A}^{-}$. The difference between them is a function of the orientation and bias errors $\left(\hat{\theta}_{\varepsilon}, \hat{\mathbf{b}}_{\varepsilon}\right)$ which the kalman filter estimates. The uncertainties of the measurements and model are expressed in terms of covariances, $\mathbf{Q}_{b}$ for bias uncertainty, $\mathbf{Q}_{\theta}$ for orientation uncertainty and $\mathbf{Q}_{Z G}, \mathbf{Q}_{Z A}$ for inclination uncertainties
4.4 Sensor model diagram. The diagram details the components of each signal $\left(\mathbf{y}_{G}\right.$ and $\left.\mathbf{y}_{A}\right)$ and the relationships between them. Gyroscope signal is modeled as the angular velocity component $\omega$, a white noise component $\mathbf{v}_{G}$ and a slowly varying gyroscope bias $\mathbf{w}_{b}$. The orientation ${ }^{G S} R$ is computed from the gyroscope signal using the box 'strapdown integration'. Acceleration is modeled as a low passed filtered white noise $\mathbf{w}_{a}$. The acceleration signal $\mathbf{y}_{A}$ is composed of the acceleration and gravity vector relative to the sensor's frame $\left({ }^{S} a-{ }^{S} g\right)$ plus a white measurement noise $\mathbf{v}_{b}$. . . . . . . . . . . . . . . . . . . . . . . . . . .


#### Abstract

4.5 Inclination estimation process. The angular velocity $\hat{\omega}_{t}^{-}$is calculated from the gyroscope signal $\mathbf{y}_{G, t}$ minus the estimated bias. Strapdown integration is then used to produce the orientation estimate ${ }^{G S} \hat{\mathbf{R}}_{t}^{-}$and inclination ${ }^{G S} \hat{\mathbf{Z}}_{G, t}^{-}$. Acceleration ${ }^{G} \hat{\mathbf{a}}_{G, t}^{-}$is calculated from the previous estimation multiplied by a factor, and then rotated to the sensor frame. Finally, the inclination estimation ${ }^{S} \hat{\mathbf{Z}}_{A, t}^{-}$is calculated from the accelerometer signal and accelerometer estimation using 4.11 . . . . .


4.6 Selecting the intersection point closest to the object location
4.7 Top level design of the system. The system is composed of two independent sub systems that estimate the location, the kalman filter estimates the errors and corrects them using a feedback loop. Two estimations of the location are used, $\hat{\mathbf{p}}_{I N S}^{-}$and $\hat{\mathbf{p}}_{R S S I}^{-}$. The difference between them is a function of the location, velocity, orientation and bias errors ( $\hat{p}_{\varepsilon}, \hat{v}_{\varepsilon}, \hat{\theta}_{\varepsilon}, \hat{\mathbf{b}}_{a, \varepsilon}, \hat{\mathbf{b}}_{g, \varepsilon}$ ) which the kalman filter estimates. The uncertainties of the measurements and model are expressed in terms of covariances, $\mathbf{Q}_{w a}$ and $\mathbf{Q}_{w g}$ for bias uncertainties, $\mathbf{Q}_{v A}$ and $\mathbf{Q}_{v G}$ for sensor white noise, $\mathbf{Q}_{v R}$ for RSSI estimation uncertainties and $\mathbf{Q}_{w r}$ for RSSI shadowing uncertainties
5.1 Experiment setup. Tracking of the hand movement across the 2D surface is accomplished by attaching several sensors to the moving hand. For inertial movement information, the SHIMMER sensor is used. For location estimation using RSSI, two BSN nodes are used as anchor nodes and one BSN node is attached to the hand. The system results are compared to reference information provided by the optical tracking system Polaris. The Polaris system uses a passive marker mounted on the moving hand.
5.2 Block diagram of the experiment data flow. Sensor data from the SHIMMER and BSN node is calibrated and pre-processed. All data sources are synchronized, and a uniform time stamp is determined. The location estimation process is composed of the INS and RSSI location estimations ( $\hat{p}_{I N S}^{-}$and $\hat{p}_{R S S I}^{-}$respectively). The kalman filter uses both estimations to predict the errors $\hat{p}_{\varepsilon}$. The INS errors are corrected in a feedback loop and the location estimation is computed $\hat{p}_{I N S}^{+}$. Polaris location estimation $\hat{p}_{\text {polaris }}$ is then compared to results and the Mean Squared Error is calculated.
5.3 The mobile node attached to the hand. A SHIMMER sensor is used for capturing INS information, a BSN node transmits data packets for RSSI measurements and a Polaris passive marker is used by the Polaris system to track the moving hand
5.4 The SHIMMER wearable sensor platform. SHIMMER incorporates a TI MSP430 processor, a CC2420 IEEE 802.15.4 radio, a triaxial accelerometer, and a rechargeable Li-polymer battery. A triaxial gyroscope board is added as an internal expansion with two dual-axis gyroscope chips. The platform also includes a MicroSD slot supporting up to 2 GB of Flash memory
5.5 The polaris tracking setup. Two passive markers are used. The first is a static marker, placed in the origin of axis defined by the RSSI tracking system. The second is a mobile marker attached to the moving hand.
5.6 Transfer function of the accelerometer sensor. $S$ is the slope of the transfer function. $V_{O F F}$ is the offset error.
5.7 Calibration sequence. The Shimmer is placed on a different face every few seconds. (a) 3-axis accelerometer sensor recording of the calibration process. The earth-facing(sky-facing) sensor is automatically detected for each position, and the average value is computed, corresponding to $g(-g)$.(b) 3-axis gyroscope sensor recording of the calibration process. A moving average method was used to detect the turns along each axis. Integrating the signal input over the complete turn corresponds to a 90 degrees turn value.
5.8 Validation sequence. The sensor placed in a different static 3 -D po-
sition every few seconds. The magnitude of the acceleration vector is
expected to be the gravity force $g$. (a) The 3 -axis accelerometer output
and the calculated norm. (b) Comparison of the magnitude to the ex-
pected gravity. In the example there are fluctuations of $0.5 \mathrm{~m} / \mathrm{s}^{2}$ which
might cause considerable errors. . . . . . . . . . . . . . . . . . . 63
5.9 Shimmer and BSN mobile node synchronization. Shimmer accelerometer and BSN accelerometer signals are shown after synchronization. Even though the BSN signal is not properly calibrated, the sync operation performs well.
5.10 Shimmer and Polaris system synchronization. Shimmer gyroscope and Polaris computed gyroscope signals are shown after synchronization. The polaris gyroscope signal is very noisy due to the derivation process, yet the sync operation works well. The shimmer gyroscope suffers from changing bias, most evident in the first 10 seconds.
6.1 Measured sensor signals during one hand movement experiment (2D movement). (a) Accelerometer output vector, only x,y axis are shown since movement is 2D. (b) Gyroscope output vector. The gyroscope changing bias can be seen in the first 10 seconds before the hand starts moving.
6.2 RSSI location error. The graph shows a changing bias with time constant of about 5 seconds. This constant is used to model the RSSI error.
6.3 Typical results for location estimations. Each plot shows the kalman-
based,RSSI-based and reference estimations. Inertial-based estimation
diverges quickly and is not presented. . . . . . . . . . . . . . . . . . 71
6.4 Typical results for location and location estimation error (LEE) decomposed for X and Y axis. Plot (a) compares the location estimation of the proposed filter with the RSSI-based estimation. The kalman filter tracks the movement and shape much better but overall RMS sense is quite similar as shown in plot (b).
6.5 Trace of the kalman gain matrix. The graph shows that the gain con-
verges after 10 seconds. . . . . . . . . . . . . . . . . . . . . . . . . . . 73
6.6 Filter tracking of gyroscope bias drift. Plot (a) compares the estimated Gyroscope bias with the actual changing bias. Plot (b) shows the Bias Estimation Error as a function of time.
6.7 Comparison of filter performance compared to simpler model without gyroscope bias model. The top plot shows the Location Estimation Error of $\mathrm{X} \operatorname{axis}\left(L E E_{x}\right)$ for the two filters. The LEE. The $L E E_{x}$ for the simple model grows due to growing error in the orientation estimation (OOE) as shown in the lower plot. The plots clearly demonstrate that the simpler model performance is inferior in face of constant Gyroscope bias
6.8 Gyroscope bias tracking during an experiment starting from a static position. The bias cannot be determined during the initial static period, it is corrected only on the 8th second when movement starts. Note the velocity trace is plotted only for reference of movement time, $y$-axis units are not relevant76
6.9 Comparison of filter tracking changing bias for different values of $\mathbf{Q}_{w g}$ [defined in 3.7]. Setting a small value, causes slow divergence
6.10 Accelerometer bias tracking. Estimated accelerometer bias values are plotted compared to the real values. The filter converges to the correct bias values for both X and Y axis.77
6.11 Location estimation for the X axis with wrong initial conditions. The plots show wrong initial X values of $0.5 \mathrm{M}, 1 \mathrm{M}$ and 1.5 M . The different traces show the filter converges after at most 20 seconds even for large errors.
6.12 Velocity estimation for the X axis with wrong initial conditions. The plots show wrong initial $X$ values of $1 \mathrm{M} / \mathrm{S}, 2 \mathrm{M} / \mathrm{S},-1 \mathrm{M} / \mathrm{S}$ and $-2 \mathrm{M} / \mathrm{S}$. The different traces show the filter converges after at most 25 seconds even for large errors.
6.13 Orientation estimation error. Orientation is represented as a counter clock-wise rotation in the XY plane. The plots show wrong initial values of 1 RAD, 2 RAD, -1 RAD and -2 RAD. For large errors, the convergence occurs only after considerable time.

## List of Tables

6.1 Kalman filter initialization parameters. ..... 70
6.2 RMSE values for each estimation method. The kalman filter achieves greater accuracy, improving the RSSI estimates by 10-20\% ..... 72
6.3 RMSE values for each $\mathbf{Q}_{w g}$ value for a typical experiment with little bias. Filter performance is affected if an inappropriate value is used. ..... 73
7.1 RMSE values for each estimation method from simulation results ..... 81

## Chapter 1

## Introduction

Recent advances in electrical, biological, chemical and mechanical sensor technologies have led to a wide range of wearable sensors suitable for long autonomous operation and continuous monitoring [2]. Using small wearable wireless platforms that can record and transmit physiological and kinematic data in real-time, human movement can now be measured continuously outside a specialized laboratory [3]. Applications in many medical fields such as gait analysis ([4],[5],[6]), functional electrical stimulation ([7], [8], [9]) and monitoring activities of daily living (ADL) ([10],[11], [12]). Thus, research is currently being carried out in many laboratories for designing systems for better and accurate tracking of human body motion with the use of on-board MEMS sensors. Beside the research efforts for high-grade, yet inexpensive sensors, advanced signal processing methods are intensely investigated to improve the performance of tracking algorithms [13].

Calculating orientation and location using these miniaturized inertial systems has limited capabilities. The main problem is that location is computed by timeintegrating the signals from gyros and accelerometers, including any superimposed sensor drift and noise. Hence, the estimation errors tend to grow unbounded.[refs] Solutions for mitigating the drift problem usually consist of an external sensors such as magnometers [14] or on problem-specific knowledge, for example gait cycle for drift error correction [15].

This paper presents a new design for mitigating the drift errors by integrating RSSI data into the estimation process. The main benefit of employing RSSI information is that it comes with no added hardware costs as all existing wireless sensor platforms support it. However, since RSSI estimation in these ranges is greatly affected by multi-path phenomenon, advanced signal processing methods must be used. We
present a complementary kalman filter model designed to optimally fuse the inertial sensors data and the RSSI data. The basic idea behind complementary filtering is that location and orientation drift errors resulting from accelerometer and gyro output errors can be bounded by aiding the estimation with additional sensor data, the information from which allows correcting the inertial sensors solution. Body segment location estimation obtained with this system was compared with a location estimation obtained using a laboratory bound camera system.

### 1.1 Motivation

Most of the research thus far has regarded the RSSI technology as unsuitable for BAN ranges due to limited accuracy and sensitivity to changing conditions. Based on Blumrozen et al. work [16] it has been shown that RSSI measurements can provide useful data when using innovative calibration techniques. This work is aimed at fusing the data from RSSI measurements with INS sensors data. Combining the sensor data from INS and RSSI can potentially mitigate for unexpected RSSI bias on the one hand and fix INS drift on the other hand. Sensor fusion is done using complementary kalman filter .

### 1.2 Thesis Contribution

This work uses kalman filter to successfully combine INS and RSSI based sensors to achieve good accuracy robust to environment changes and drift. The achieved accuracy is in the scale of centimeters and improves previous results by 20-80 percent. The whole system is still very low cost and is very light and small. Real world experiments show that the sensor fusion achieves greater accuracy even in the face of sensor bias and environment changes. Since almost all wireless devices can sample RSSI data, this method can be utilized to improve many wireless INS systems used for close range.

To our knowledge, this work is the first to combine RSSI and INS sensors data to track human body motion. Further more we conducted thorough simulation testing and real world experiments to verify results and robustness of the system.

The second contribution is presented in 4.1.2, an alternative derivation for the orientation estimation kalman filter. The resulting model is simple and easier to
implement.

### 1.3 Thesis Outline

The thesis is organized as the following.
Chapter 2 introduces the related work that was done in this area. The system model is presented in Chapter 3. Chapter 4 presents the problem formally and describes the solution

Chapter 5 deals with the experimental methods and setup. The experiments results and analysis are provided in chapter 6. Chapter 7 provides a through discussion of the results and future work. Finally, Chapter 8 summarizes the presented work and outlines the conclusions.

## Chapter 2

## Related Work

In recent years, technological advancements have made it possible to record human movement continuously outside the boundaries of the laboratory [17]. Wearable wristsize sensor platform that can record inertial movement are now commercially available. The SHIMMER [3] and XSENS [18] platforms are small wireless sensor platforms that can record and transmit physiological and kinematic data in real-time and are widely used for medical research. The UP wristband by Jawbone [19] is a wireless wristband aimed for the general public, collecting every day life statistics such as sleep time and steps count. As these technologies are now widespread, the topic of human motion tracking and gait analysis is of great interest in everyday applications generating a large amount of research into systems that can provide this information in real time at a low-cost and with the smallest intrusion level possible [20].

### 2.1 Inertial motion tracking

Inertial navigation systems (INSs) were widely used for ships, submarines, and airplanes starting from the 1950s [21]. Over the course of the last 20 years, developments in fields of electronics and micromachining, pushed by the needs of the automotive and consumer industry have produced low-cost, small sized silicon sensors leading to new applications [17]. Inertial sensors have been extensively used in the automotive industry [22], robotics [23] and augmented reality [24]. In recent years, they have also applied into the implementation of human motion tracking [20].

Inertial trackers are of prime importance in human motion tracking. They have fewer costs, compact size, lightweight, and no motion constraint. They are completely self-contained, so they have no Line Of Sight (LOS) requirements, no emitters
to install, and no sensitivity to interfering electromagnetic fields or multipath effects. Also, they have very low latency and can be measured at relatively high rates (thousands of samples per second) [25]. Sadly, they have a major drawback that is hard to overcome, the drift problem. There are several causes of drift in a system which obtains orientation by integrating the outputs of angular rate gyros [26]:

- Constant bias $\delta \omega$ - when integrated causes a steadily growing angular error $\theta_{t} \delta \omega \cdot t$
- Thermo-Mechanical White Noise $v_{t}$ - when integrated leads to a random walk process $\theta_{t}=\int_{0}^{t} v_{t} d t$ which has an expected value zero but a mean squared error growing linearly with time.
- Calibration errors in the scale factors, alignments, and linearities of the sensors, produce measurement errors leading to the accumulation of additional drift.
- Flicker Noise / Bias Stability - during operation the bias wanders away, producing a residual bias that gets integrated to create second-order random walk. Bias stability is usually modeled as a random walk or Gauss-Markov process, and is often the critical parameter for drift performance, since constant bias can usually be calibrated and compensated effectively.

For position estimation the drift problem is even more severe. First, there are accelerometer errors corresponding to the 4 gyro errors listed above. However, since the position is obtained by double integrating acceleration, a fixed accelerometer bias error results in a position drift error that grows quadratically in time. But the critical cause of error in position measurement is error in the orientation estimation. An error of $\delta \theta$ in tilt angle will result in an error of $g \cdot \sin (\delta \theta)$ in the horizontal components of the acceleration. For this reason, in practice, it is the gyroscopes, not the accelerometers which limit the positional navigation accuracy of most INS systems. As seen in fig 2.1, the simulation shows that commercial-grade inertial navigation systems suffer from large position drift after just a few seconds [1]. Therefore, exploring new techniques to reduce drift in inertial systems is a primary goal in the field of human motion tracking.


Figure 2.1: Comparison of position drift performance of commercial, tactical, navigation, strategic-grade, and perfect pure inertial navigation systems [1]. Note that commercial and tactical grade systems are rendered useless after just a few seconds. Also, even the "perfect" ins eventually drifts due to fluctuations in earth's gravitational field.

### 2.2 Reducing drift in inertial systems

Many studies of reducing drift in inertial systems for human motion tracking have been performed. Different systems with varying number, type and configuration of sensors used, some studies are limited to tracking two degrees of orientation in a plane, while others track 3-D orientation [27]. Algorithms have also been designed to track limb segment orientations relative to each other or calculate joint angles [28]. Generally, the solutions for the drift reduction problem fall into one of the two categories, sensor fusion and the application of domain specific assumptions.

### 2.2.1 sensor fusion

Sensor fusion is the process of combining sensory data from two or more types of sensors to update the state of a system. In inertial systems, the state usually consists of
position, velocity and orientation of the object. A sensor fusion algorithm calculates the state using inertial sensors signals together with signals from additional sources such as magnetometers [29]. Several mathematical and statistical tools are employed in current research to fuse sensor data for tracking. The most prominent methods are particle filter and various variants of the kalman filter , mainly EKF, unsentenced kalman and complementary kalman filter. Many types of complementing sensor technologies are found in the literature, among them optical [30], Ultra Wide Band [31], GPS [32], magnetometer [28], and acoustical sensors [33].

The integration of visual and inertial sensors for human motion tracking has received much attention lately, due to its robust performance and wide potential application [34]. Most systems use an extended kalman filter for sensor fusion. The most popular representant is the commercially available InterSense VIS-Tracker [35]. In [30] a real-time hybrid solution is presented to articulated 3D arm motion tracking for home-based rehabilitation by combining visual and inertial sensors. Data fusion is also done using Extended Kalman Filter.In [36] the FlightTracker system was presented to track a pilot's head movement. They developed a differential inertial measurement equation of the pilots head motion relative to the aircraft, which can then have its drift corrected by periodic optical measurements of the head position relative to the aircraft. Inertial sensors can provide cues about the observed scene structure. This information can be used to simplify 3D reconstruction of the observed world. Lobo and Dias [37] use inertial sensors to find a vertical reference and use it to determine the image horizon line. Similar work has been described in these refs [38], [39].

Another type of sensor commonly used to reduce drift is the vector magnetometer. The magnetometers measure the strength and direction of the local magnetic field, allowing the north direction to be found. Magnetometers are susceptible to interference of ferromagnetic materials which distort the orientation measurement, therefor they are not accurate enough to replace gyroscopes [ref]. However, they can be used efficiently together with gyroscope data to improve the accuracy of the calculated orientation. Foxlin [40] and Bachman [28] presented filters in which accelerometers and magnetometers are used for low frequency components of the orientation and gyroscopes to measure faster changes in orientation. In [29] a complementary kalman filter was designed to overcome ferromagnetic disturbances by calculating a magnetic disturbance vector. The main advantage of this approach is that the tracking system remains self contained. The main disadvantage is that it only reduces the drift growth
rate, rather than allowing absolute corrections to be applied.

### 2.2.2 Domain specific assumptions

In some applications, assumptions of the movement of the body can be made and used to reduce drift. One of the best examples for using domain specific assumptions is NavShoe [41] where a shoe mounted IMU is used for pedestrian tracking. When a person walks, their feet alternate between a stationary stance phase and a moving stride phase. The system uses the stance phase for zero velocity updates, allowing drift in velocity to be corrected. By measuring the acceleration due to gravity during the stationary phase, inclination errors can also be corrected. The work of Yun [42] also uses the periodic nature of walking for precise drift error correction. In [43], Luinge presented a complementary kalman filter for human body segment orientation fusing gyroscope and accelerometer data. The system models the characteristic acceleration of body segments as a first order low-pass filtered white noise process. In similar work, Bachman [44] present a quaternoin based kalman filter for inertial tracking. The filter continuously corrects the drift based on the assumption that human limb acceleration is bounded, and averages to zero over any extended period of time. Another approach for domain specific assumptions is the use of kinematic constraints model.Young [45] demonstrated a method for estimating the linear acceleration of IMUs based on subject body model constraints. Zhou and Hu [46] developed a kinematic model for human upper limb movement which helps at removing undesirable biases or noise. Using this model, they construct a kalman filter which fuses inertial sensors data to track human movement.

The work in this research employs both techniques for drift reduction. First, domain specific assumption is employed in the orientation estimation. The limb acceleration is assumed to be small compared to the gravity which allows the filter to continuously obtain an inclination estimate using the signal of the 3D acceleromter. This estimate is then used by a complementary kalman filter to continuously reduce the gyroscope drift [43]. Second, sensor fusion is employed to optimally combine the inertial system estimations with RSSI-based position estimations. The sensor fusion is also implemented using a complementary kalman filter.

### 2.3 RSSI-based tracking

RSSI-based tracking systems are mainly used in the context of indoor localization. In general, indoor localization algorithms are aimed at locating wirelessly an object or a person inside a building [47]. They assume the presence of a limited number of reference nodes, referred to as anchor nodes, which know their own coordinates and are used as reference points for localizing the other nodes [48]. RSSI-based tracking systems use Received Signal Strength Indication (RSSI) to get an estimate of the distance between transmitter and receiver (ranging) [49]. Sadly, the indoor radio channel is very unpredictable, since reflections of the signal from furniture, walls, floor and ceiling may result in severe multipath interference at the receiving antenna. This results in range estimation errors which translate to large positioning errors [50]. Nonetheless, RSSI-based algorithm are intensively studied for indoor tracking due to many advantages over other physical layer measurements such as acoustic Time Of Arrival systems [51]. Compared to other solutions, pure RSSI methods are low cost, low energy and can be easily deployed in wireless sensor network platforms, since RSSI data is natively supported by most of the existing transceiver chipsets, with no extra hardware costs [48].

RSSI-based localization algorithms are mostly in one of two categories, rangebased and range-free algorithms. Range-based algorithms use the RSSI signal to estimate the distance between nodes. Then, the object position is estimated using several techniques such as triangulation and trilateration [52],[53]. This method is susceptible to multipath and shadowing effects and thus has limited accuracy. Rangefree algorithms do not use the RSSI value to compute distance, but exploit it in other ways. In RSSI mapping technique, the RSSI value is interpreted using a precalculated RSSI map. Systems such as RADAR [54] require a preliminary accurate mapping of RSSI values in each position on the map. Comparing the RSSI values received from the different anchors with the pre-built RSSI map, a node can estimate its own position in the area.

RSSI-based tracking systems accuracy is in the scale of a meter [55]. Therefor RSSI was not considered a viable solution for the human motion tracking problem which requires a much better accuracy. In recent work, Blumrozen et al. [56] suggested that under certain condition, RSSI can be a valid solution for the human motion tracking problem. Using anchor nodes in close proximity, transmitting in high frequencies, the RSSI-mapping based system achieved accuracy of several centimeters. In [16],
the work is extended with a more elaborate calibration scheme, advanced processing techniques and additional real life experiments. Since RSSI is readily deployable with no extra hardware needed, it is a prime candidate to be used in sensor fusion with an inertial system. This research uses a complementary kalman filter to fuse the described RSSI system with inertial system's data to solve the human tracking problem.

Fusing RSSI and inertial sensors' data has been previously done only in the context of indoor localization. In [57] Klingbeil presents a modular framework for fusion of many types of sensory input, among then RSSI and inertial tracking. The sensor fusion is done using particle filter. In [58] Fink presents an RSSI-based system which uses diversity and sensor fusion of RSSI and inertial sensors to achieve must better precision. The diversity concept is using RSSI with redundant data transmission in different frequency bands which can reduce the dropout probability. In another work, Evennou et al. [59] presents a system for pedestrian localizations by means of sensor fusion of WiFi signal strength measurments with inertial sensors signals. A structure based on a Kalman filter and a particle filter is proposed.

## Chapter 3

## System Model

### 3.1 System description

The system components are illustrated in Figure 3.1. The system consists of a single mobile node and $N$ static nodes, referred to as anchor nodes. Each node is equipped with wireless transceiver that can periodically transmit data. The mobile node transmits a data packet with a known transmission power to the anchor nodes. The anchor nodes, located in the transmission range of the mobile node, calculate the received power values (RSSI). The mobile node is also equipped with inertial sensors (miniature gyroscope and accelerometer) which record the node's inertial data. The goal of this work is to continuously estimate the mobile node's location from the mobile node inertial data and from the attenuation of the electromagnetic signal in space as measured by the anchor nodes.

The research is aimed at solving the location estimation problem in the realm of human body movement. Hence, the mobile node is assumed to be attached to a human body part. This restriction is used to design a solution tailored for the BAN environment, greatly improving results.

### 3.2 Inertial sensors modeling

The system is composed of a 3D accelerometer sensor and a 3D gyroscope sensor. Other types of sensors, such as magnetic compass, can be possibly combined and used in the future to further improve results [28]. Figure 3.2 depicts the inertial sensors model as described in detail in the following sections.


Figure 3.1: System description diagram. The diagram details the system's components, the mobile node, the anchor nodes and the control station and the relationship between them. The mobile nodes periodically sends a data packet to the anchor nodes, which compute the Recieved Signal Stregnth. The control station aggregates, synchronizes and estimates location utilizing both inertial and RSS-based estimations

### 3.2.1 Rotations and Sensor co-ordinate system

Let us denote the location, velocity and acceleration of the object $c$ in the global co-ordinate system by ${ }^{G} \vec{p}_{c},{ }^{G} \vec{v}_{c},{ }^{G} \vec{a}_{c}$, respectively. The superscripts $G$ and $S$ are used to denote vectors that are expressed in the global and sensor co-ordinate systems, respectively, i.e. ${ }^{S} \vec{a}_{c}$ is the object's acceleration in the sensor frame.

The orientation of the sensor with respect to the global co-ordinate system is expressed with a rotation matrix containing the three unit vectors of the global coordinate system expressed in the sensor frame [43].

$$
{ }^{G S} \mathbf{R}=\left[\begin{array}{lll}
{ }^{S} \mathbf{X} & { }^{S} \mathbf{Y} & { }^{S} \mathbf{Z} \tag{3.1}
\end{array}\right]^{T}
$$



Figure 3.2: Sensor model diagram. The diagram details the components of each signal $\left(\mathbf{y}_{G, t}\right.$ and $\left.\mathbf{y}_{A, t}\right)$ and the relationships between them. Gyroscope signal is modeled as the angular velocity component $\omega_{t}$, a white noise component $\mathbf{v}_{G}$ and a slowly varying gyroscope bias $\mathbf{w}_{b g}$. The orientation ${ }^{G S} R$ is computed from the gyroscope signal using the box 'integration'. The acceleration signal $\mathbf{y}_{A, t}$ is composed of the acceleration and gravity vector relative to the sensor's frame $\left({ }^{S} a-{ }^{S} g\right)$, a white measurement noise $\mathbf{v}_{A}$ and a slowly varying accelerometer bias $\mathbf{w}_{b a}$.

The rotation matrix is a linear transformation to the global co-ordinate system

$$
{ }^{G} \overrightarrow{\mathbf{a}}_{c}={ }^{G S} \mathbf{R} \cdot{ }^{S} \overrightarrow{\mathbf{a}}_{c} .
$$

Transformations can also be expressed in terms of rotation vector $\vec{\theta}$. The rotation is expressed as a single rotation along the axis of the vector.

$$
\vec{\theta}=\theta \cdot \hat{\mathbf{e}} .
$$

Where $\hat{\mathbf{e}}=\left[\begin{array}{lll}e_{X} & e_{Y} & e_{Z}\end{array}\right]^{T}$ is a three dimensional unit vector representing the axis of turn and $\theta$ is a scalar representing the angle of rotation. This representation is very useful since it expresses a rotation in only 3 scalars, thus simplifying the kalman state. Another useful property is that gyroscope sensor readings are easily expressed in this form. If $\mathbf{T}$ is the time sample, and $\omega_{t}$ is the angular velocity, the rotation
can be represented as a rotation vector $\mathbf{T} \omega_{t}$. Further more, for small rotations, the relationship between the original vector and the rotated vector is given by [60]

$$
\begin{equation*}
{ }^{G} v={ }^{S} v\left(\mathbf{I}+\left[{ }^{G S} \theta \times\right]\right) . \tag{3.2}
\end{equation*}
$$

Where ${ }^{G S} \theta \times$ is a small rotation vector from $S$ to $G$. This equation also applies to the orientation matrices, yielding the following property

$$
\begin{equation*}
{ }^{\text {new }} R={ }^{\text {old }} R(\mathbf{I}+[\theta \times]) \tag{3.3}
\end{equation*}
$$

### 3.2.2 The Gyroscope sensor model

The gyroscope signal is modeled [61] as a combination of white measurement noise $\mathbf{v}_{G, t}$, gyroscope bias $\mathbf{b}_{t}$ and the angular velocity $\omega_{t}$

$$
\begin{equation*}
\mathbf{y}_{G, t}=\omega_{t}+\mathbf{b}_{g t}+\mathbf{v}_{G, t} . \tag{3.4}
\end{equation*}
$$

Where $\mathbf{v}_{G, t}$ is a white noise process with covariance $\mathbf{Q}_{v G}$ [43].

$$
\begin{equation*}
\mathbf{v}_{G, t} \sim \mathbf{N}\left(0, \sigma_{g, t}^{2}\right) \tag{3.5}
\end{equation*}
$$

The gyroscope bias is modeled as a slowly varying signal [62]. The bias fluctuations are caused by changing properties of the sensor (such as temperature, mechanical wear and battery power level). The model is a first order Markov process, driven by a small white Gaussian noise.

$$
\begin{equation*}
\mathbf{b}_{g, t}=\mathbf{b}_{g, t-1}+\Delta t \mathbf{w}_{b g, t} . \tag{3.6}
\end{equation*}
$$

Where $\Delta T$ is the time step and $\mathbf{w}_{b g, t}$ is a white noise process with covariance $\mathbf{Q}_{w g}$.

$$
\begin{equation*}
\mathbf{w}_{b, t} \sim \mathbf{N}\left(0, \sigma_{w g, t}^{2}\right) \tag{3.7}
\end{equation*}
$$

The equivalent formula in the continuous mode yields

$$
\begin{equation*}
\dot{\mathbf{b}}_{g, t}=\mathbf{w}_{b g, t} . \tag{3.8}
\end{equation*}
$$

Where $\dot{\mathbf{b}}$ is the derivative of $\mathbf{b}$.

### 3.2.3 The Accelerometer sensor model

The accelerometer signal is modeled as the sum of the acceleration vector ${ }^{S} \overrightarrow{\mathbf{a}}_{t}$, the gravity vector, small sensor bias $\mathbf{b}_{a, t}$ and white gaussian measurement noise $\mathbf{v}_{A, t}$ [43]. All vectors are in the sensor's reference frame.

$$
\begin{equation*}
\mathbf{y}_{A, t}={ }^{S} \mathbf{a}_{t}-{ }^{S} \mathbf{g}_{t}+\mathbf{b}_{a, t}+\Delta t \mathbf{v}_{A, t} . \tag{3.9}
\end{equation*}
$$

Where $\mathbf{v}_{A, t}$ is a white noise process with covariance $\mathbf{Q}_{v A}$.

$$
\begin{equation*}
\mathbf{v}_{A, t} \sim \mathbf{N}\left(0, \sigma_{A, t}^{2}\right) \tag{3.10}
\end{equation*}
$$

The accelerometer bias is modeled similarly to the gyroscope bias model [62]

$$
\begin{equation*}
\mathbf{b}_{a, t}=\mathbf{b}_{a, t-1}+\Delta t \mathbf{w}_{b a, t} . \tag{3.11}
\end{equation*}
$$

Where $T$ is the time step and $\mathbf{w}_{b a, t}$ is a white noise process with covariance $\mathbf{Q}_{w a}$.

$$
\begin{equation*}
\mathbf{w}_{b a, t} \sim \mathbf{N}\left(0, \sigma_{w a, t}^{2}\right) \tag{3.12}
\end{equation*}
$$

The equivalent formula in the continuous mode yields

$$
\begin{equation*}
\dot{\mathbf{b}}_{a, t}=\mathbf{w}_{b a, t} . \tag{3.13}
\end{equation*}
$$

### 3.3 RSSI modeling

Received Signal Strength Indicator (denoted RSSI) is an indication of the power level received by the antenna. In the context of this research, RSSI measurements are performed between the mobile node and several anchor nodes (see Section 3.1). The antennas are assumed to be omnidirectional. The RSSI signal is affected by the changing properties of the signal dispersion due to reflections and other multi path phenomenon. The received signal power at the anchor node is given by the formula [63]

$$
\begin{equation*}
L r_{i}^{t}=L t+A-q 10 \log _{10} d_{i}^{t}+\alpha^{t} \tag{3.14}
\end{equation*}
$$

Where $L t$ is the transmission power, $L r_{i}^{t}$ is the received signal power at node $i, d_{i}^{t}$ is the distance between the anchor node $i$ and the mobile node, $A$ is a constant power
offset which is determined by several factors, like receiver and transmitter antenna gains and transmitter wave length , $q$ is the channel exponent which varies between 2 (free space) and 4 (indoor with many scatterers), and $\alpha^{t}$ is a Gaussian distributed random variable with zero mean and standard deviation $\sigma_{\alpha}$ that accounts for the random effect of shadowing. Note that $\alpha^{t}$ may be correlated between successive measurements, due to temporal shadowing phenomenon.

Combining RSSI data from several anchor nodes, allows for exact location estimation $\hat{\mathbf{p}}_{t}$ using triangulation or similar methods (see [16]).

$$
\hat{\mathbf{p}}_{t}=f\left(\left[L r_{i}^{t}\right]_{i=1}^{N}\right) .
$$

The RSSI based location estimation $\hat{\mathbf{p}}_{t}$ is affected by the changing properties of the signal dispersion due to reflections and other multi path phenomenon. It can be modeled as a sum of the actual position, slowly varying shadowing factor, and an instantaneous white Gaussian noise:

$$
\begin{equation*}
\hat{\mathbf{p}}_{t}=\mathbf{p}_{t}+\mathbf{b}_{r, t}+\mathbf{v}_{R, t} \tag{3.15}
\end{equation*}
$$

Where $\mathbf{p}_{t}$ is the real location, $\mathbf{b}_{r, t}$ is the bias error and $\mathbf{v}_{R, t}$ is a simple white noise process with covariance $\mathbf{Q}_{v R}$.

$$
\begin{equation*}
\mathbf{v}_{R, t} \sim \mathbf{N}\left(0, \sigma_{r, t}^{2}\right) \tag{3.16}
\end{equation*}
$$

The bias $\mathbf{b}_{r, t}$ is related to shadowing phenomenon and to directionality of the antenna. The bias is correlated between successive measurements, due to large scale fading factor in the $\alpha^{t}$ component from the Formula 3.14. The correlation caused by deflections from objects can be modeled as a first order Gauss-Markov process [64] with auto correlation function of

$$
\mathbf{R}_{b r}(t)=\mathbf{E}\left(\mathbf{b}_{r, t^{\prime}}, \mathbf{b}_{r, t^{\prime}+t}\right)=\sigma_{b r}^{2} e^{-\frac{|t|}{\tau}}
$$

Where $\sigma_{b r}^{2}$ is the variance and $\tau$ is the correlation time of the interference that could be due to a reflecting object and other multi path phenomenon. Notice that the correlation time $\tau$ is dependent of the movement speed and path properties. Experiments data shows that $\tau$ should be in the range of $0.5-1$ seconds for most human motion scenarios.

The Gauss-Markov process can be represented in state space representation ([65],[61])
as follows

$$
\begin{equation*}
\mathbf{b}_{r, t}=e^{-\frac{|\Delta t|}{\tau}} \mathbf{b}_{r, t-1}+\Delta t \mathbf{w}_{r, t} . \tag{3.17}
\end{equation*}
$$

Where $\tau$ is the time constant, $\Delta t$ is the discrete time step and $\mathbf{w}_{r, t}$ white noise process with the covariance $\mathbf{Q}_{w r}$

$$
\begin{equation*}
\mathbf{w}_{r, t} \sim \mathbf{N}\left(0, \sigma_{b r, t}^{2}\right) \tag{3.18}
\end{equation*}
$$

For continuous time systems, the following formula is used [66]

$$
\dot{\mathbf{b}}_{r t}=-\frac{1}{\tau} \mathbf{b}_{r t}+\sqrt{2 \sigma^{2}} u(t)
$$

The state space representation of the noise model is of significant importance since kalman filter inputs must be in that form. More specifically, kalman filter can be extended to work with colored noise, only if the noise can be represented in the state space representation [67].

## Chapter 4

## Location estimation

This chapter explores the problem of optimal location estimation. Section 4.1 solves the estimation problem when only inertial data is used. Then , section 4.2 solves the estimation problem when only RSSI data is available. Finally, section 4.3 presents a solution for estimating location using both data sources using advanced sensor fusion methods. The sensor fusion is accomplished using a complementary kalman filter, constantly estimating errors and sending feedback to the INS system.

### 4.1 Location estimation using inertial sensors

This section explores the problem of optimally estimating location using inertial sensors data only. The IMU (Inertial Measurement Unit) is composed of 3D accelerometer and gyroscope systems which are strapped to a human body part, constantly recording the movement as presented in Section 3.1.

The system design is depicted in Figure 4.1. Section 4.1.1 details the operation of the basic INS component, which integrates the sensors' data to compute a location estimate. As previously explained in Section 2.1, INS systems suffer greatly from integration drift and sensor bias. These error factors are so acute that they render the estimation useless after a short time. Section 4.1.2 derives a kalman filter for orientation estimation that can improve the inertial system's results. It uses domainspecific assumption of human motion to estimate inclination using accelerometer data and compares it to the INS estimation. The difference between the two inclination estimations is then used to estimate orientation and gyroscope bias errors. The system uses a feedback design, to constantly correct INS estimations according to the kalman filter corrections.


Figure 4.1: Top level design of the inertial sensors based estimation system. The two inertial sensor systems, 3D accelerometer and 3D gyroscope send their data to the INS component ( $\mathbf{y}_{A, t}$ and $\mathbf{y}_{G, t}$ respectively). Integrating the data, the INS main component estimates the location, velocity and orientation ( $\hat{\mathbf{p}}_{i n}, \hat{\mathbf{v}}_{i n}$ and $\hat{\theta}_{\text {in }}$ respectively). An orientation kalman filter is used to improve robustness of the system the face of gyroscope bias. The kalman filter uses orientation and acceleration data ( $\hat{\theta}_{i n, t}$ and $\mathbf{y}_{A, t}$ respectively) to estimate orientation and bias errors $\left(\theta_{\varepsilon}, b_{g, \varepsilon}\right)$. These errors are used in a feedback loop by the INS component to correct future estimations. The kalman filter uses the covariance matrices of the accelerometer white noise, gyroscope white noise and gyroscope bias noise to optimally estimate errors ( $Q_{v A}, Q_{v G}, Q_{w g}$ )

### 4.1.1 Strapdown inertial navigation

The basic algorithm for strap down inertial navigation system is displayed in Figure 4.2 , this section describes the algorithm in detail. The first paragraph, explains orientation tracking. Relying on these results, position estimation is outlined on the second part of this section.

Tracking orientation The expression for the gyroscope bias estimation is trivially derived from the bias model Equation (3.6), setting noise factors to 0 , the estimation is a constant

$$
\begin{equation*}
\mathbf{b}_{i n, g t}=\mathbf{b}_{i n, g(t-1)} . \tag{4.1}
\end{equation*}
$$

The estimated angular velocity $\omega_{i n, t}$ is derived from Equation (3.4) by solving for


Figure 4.2: Strapdown intertial navigation algorithm. The angular velocity $\hat{\omega}_{i n, t}$ is calculated from the gyroscope signal $\mathbf{y}_{G, t}$ minus the estimated bias. Strapdown integration is then used to produce the orientation estimate ${ }^{G S} \hat{\mathbf{R}}_{t}$. Acceleration is calculated from the accelerometer signal $\mathbf{y}_{A, t}$ minus the estimated bias, rotated to the global frame minus the gravity vector. Finally, the velocity ${ }^{G} \hat{v}_{t}$ is found by integrating the acceleration ${ }^{G} \hat{a}_{t}$, and location ${ }^{G} \hat{p}_{t}$ is found by integrating the velocity.
the term $\omega_{i n, t}$ and setting the error factor to 0 .

$$
\omega_{i n, t}=\mathbf{y}_{G, t}-\mathbf{b}_{i n s, g t} .
$$

The orientation of an Inertial Measurement Unit (IMU) relative to the global coordinate system is tracked by 'integrating' the angular velocity signal $\omega_{i n, t}=\left(\omega_{i n X, t}, \omega_{i n Y, t}, \omega_{i n Z, t}\right)^{T}$ obtained from the system's rate-gyroscopes.

The body orientation can be described by an orientation matrix ${ }^{G S} \mathbf{R}$ as defined in Equation (3.1). To track the orientation of an IMU we must track ${ }^{G S} \mathbf{R}$ through time. It the orientation at time $t$ is given by $\mathbf{R}_{t}$ then the rate of change of $\mathbf{R}$ at $t$ is given by [68]

$$
\begin{equation*}
\dot{\mathbf{R}}_{t}=\lim _{\Delta t \rightarrow 0} \frac{\mathbf{R}_{t+\Delta t}-\mathbf{R}_{t}}{\Delta t} \tag{4.2}
\end{equation*}
$$

The matrix $\mathbf{R}_{t+\Delta t}$ can be rewritten using the Equation (3.2)

$$
\mathbf{R}_{t+\Delta t}=\mathbf{R}_{t}(I+[\psi \times])
$$

Where the small angle approximation holds since $\Delta t$ is small. Substituting the expression for $\mathbf{R}_{t+\Delta t}$ in the Equation (4.2), yields

$$
\begin{aligned}
\dot{\mathbf{R}}_{t} & =\lim _{\Delta t \rightarrow 0} \frac{\mathbf{R}_{t}(I+[\psi \times])-\mathbf{R}_{t}}{\Delta t} \\
& =\mathbf{R}_{t} \lim _{\Delta t \rightarrow 0} \frac{[\psi \times]}{\Delta t}
\end{aligned}
$$

In the limit $\Delta t \rightarrow 0$, the following property holds

$$
\lim _{\Delta t \rightarrow 0} \frac{[\psi \times]}{\Delta t}=\left[\omega_{t} \times\right]
$$

Where $\omega_{t}$ is the angular velocity at time $t$. Hence, the orientation is given by the solution to the differential equation

$$
\dot{\mathbf{R}}_{t}=\mathbf{R}_{t}\left[\omega_{t} \times\right] ;
$$

which has the solution

$$
\mathbf{R}_{t}=\mathbf{R}_{0} \cdot \exp \left(\int_{0}^{t}\left[\omega_{t} \times\right] d t\right)
$$

If the orientation is represented using rotation vectors, the solution is much simpler. Let us denote $\theta_{t}$ as the rotation vector representing the orientation at time $t$. The angular velocity can be defined as the derivative of the rotation vector

$$
\begin{equation*}
\dot{\theta}_{t}=\omega_{t} . \tag{4.3}
\end{equation*}
$$

Thus, the expression for orientation is simply given by

$$
\begin{equation*}
\theta_{t}=\int_{0}^{t} \omega_{t} d t \tag{4.4}
\end{equation*}
$$

The strap down algorithm, uses the last result to calculate the orientation estimation

$$
\begin{align*}
& \theta_{i n, t}=\theta_{i n, 0} \int_{0}^{t} \omega_{i n, t} d t \\
& \dot{\theta}_{i n, t}=\omega_{i n, t} . \tag{4.5}
\end{align*}
$$

Tracking position To track the position of the INS, the acceleration estimation in the global co-ordinate system is derived. The expression for the bias estimation is trivially derived from the bias model Equation (3.11), setting noise factors to 0 , the estimation is a constant

$$
\begin{equation*}
\mathbf{b}_{i n, a t}=\mathbf{b}_{i n, a(t-1)} \tag{4.6}
\end{equation*}
$$

Equation (3.9) yields the following property

$$
\mathbf{y}_{A, t}-\mathbf{b}_{i n, a t}={ }^{S} \mathbf{a}_{i n, t}+{ }^{S} \mathbf{g}_{t} .
$$

Using the resulting expression and projecting it into the global frame of reference, yields

$$
{ }^{G S} \mathbf{R}_{t} \cdot\left({ }^{S} \mathbf{y}_{A, t}-\mathbf{b}_{i n, a t}\right)={ }^{G} \mathbf{a}_{i n, t}+{ }^{G} \mathbf{g}_{t}
$$

Subtracting the gravity vector ${ }^{G} \mathbf{g}_{t}$ from the above expression results with the acceleration estimation.

$$
\begin{equation*}
{ }^{G} \mathbf{a}_{i n, t}={ }^{G S} \mathbf{R}_{t} \cdot\left({ }^{S} \mathbf{y}_{A, t}-\mathbf{b}_{i n, a t}\right)-{ }^{G} \mathbf{g}_{t} \tag{4.7}
\end{equation*}
$$

The acceleration is integrated once to obtain velocity, and again to obtain displacement:

$$
\begin{aligned}
{ }^{G} \mathbf{v}_{i n, t} & =\int_{0}^{t}{ }^{G} \mathbf{a}_{i n, t} d t \\
{ }^{G} \mathbf{p}_{i n, t} & =\int_{0}^{t}{ }^{G} \mathbf{v}_{i n, t} d t
\end{aligned}
$$

Equivalently, these properties hold

$$
\begin{align*}
{ }^{G} \dot{\mathbf{v}}_{i n, t} & ={ }^{G} \mathbf{a}_{i n, t} ;  \tag{4.8}\\
{ }^{G} \dot{\mathbf{p}}_{i n, t} & ={ }^{G} \mathbf{v}_{i n, t} . \tag{4.9}
\end{align*}
$$

### 4.1.2 Domain-specific assumptions - the orientation kalman filter

In some applications it is possible to make assumptions about the movement of the body to which the IMU is attached [41]. Such assumptions can be used to minimize drift. The kalman filter presented in this section uses assumptions of human body motion to detect orientation drift errors. The filter is based on the work of Luinge et al. [43] which presents an optimum filter for measuring human body segments orientation using INS sensors. The filter overcomes gyroscope drift problems by correcting the estimation using the acceleration data.

This section contains a detailed overview of the kalman filter. We present a different derivation than specified in [43], the resulting matrices are much simpler while preserving performance. The following is a concise description of the derivation, please refer to the original paper for more details.

## Sensor fusion using kalman filter

Orientation is estimated using a complementary kalman filter, using 3D accelerometer and 3D gyroscope systems. The structure of the kalman filter is shown in Figure 4.3. Each of the sensor systems is used to derive an independent estimation of the inclination, with different accuracy and error sources( $\hat{\mathbf{Z}}_{A}^{-}$for accelerometer and $\hat{\mathbf{Z}}_{G}^{-}$ for gyroscope). The difference between the estimations ( $\hat{\mathbf{Z}}_{A}^{-}-\hat{\mathbf{Z}}_{G}^{-}$) is modeled as a function of errors in the measurements, specifically gyroscope orientation error, gyroscope bias error and measurements noise. This function is the error model which ,when converted to state space format, serves as the basis for the kalman filter.

## Model of sensor signals

The sensor unit is attached to a human body, hence the model is based on characteristics of human motion. The main assumption states that the acceleration of a body segment in the global reference frame can be described as a low pass filtered white noise. It relies on the fact that the body segment can't be going in the same direction for too long, it will hit a wall after some time.

The gyroscope signal and bias are modeled as described in Equation (3.4) and Equation (3.6). The accelerometer signal is modeled as the sum of the acceleration vector ${ }^{S} \overrightarrow{\mathbf{a}}_{t}$, the gravity vector and white gaussian measurement noise $\mathbf{v}_{A, t}$. All vectors


Figure 4.3: Structure of the kalman filter. Two estimations of the inclination are used, $\hat{\mathbf{Z}}_{G}^{-}$and $\hat{\mathbf{Z}}_{A}^{-}$. The difference between them is a function of the orientation and bias errors $\left(\hat{\theta}_{\varepsilon}, \hat{\mathbf{b}}_{\varepsilon}\right)$ which the kalman filter estimates. The uncertainties of the measurements and model are expressed in terms of covariances, $\mathbf{Q}_{b}$ for bias uncertainty, $\mathbf{Q}_{\theta}$ for orientation uncertainty and $\mathbf{Q}_{Z G}, \mathbf{Q}_{Z A}$ for inclination uncertainties


Figure 4.4: Sensor model diagram. The diagram details the components of each signal $\left(\mathbf{y}_{G}\right.$ and $\left.\mathbf{y}_{A}\right)$ and the relationships between them. Gyroscope signal is modeled as the angular velocity component $\omega$, a white noise component $\mathbf{v}_{G}$ and a slowly varying gyroscope bias $\mathbf{w}_{b}$. The orientation ${ }^{G S} R$ is computed from the gyroscope signal using the box 'strapdown integration'. Acceleration is modeled as a low passed filtered white noise $\mathbf{w}_{a}$. The acceleration signal $\mathbf{y}_{A}$ is composed of the acceleration and gravity vector relative to the sensor's frame $\left({ }^{S} a-{ }^{S} g\right)$ plus a white measurement noise $\mathbf{v}_{b}$
are in the sensor's reference frame.

$$
\mathbf{y}_{A, t}={ }^{S} \mathbf{a}_{t}-{ }^{S} \mathbf{g}_{t}+\mathbf{v}_{A, t}
$$

The acceleration vector is modeled as a first order low-pass filtered white noise process. The model is based on the assumption that during human motion, the acceleration vector changes constantly and is much smaller than the gravity vector.

$$
\begin{equation*}
{ }^{G} \mathbf{a}_{t}=c_{a} \cdot{ }^{G} \mathbf{a}_{t-1}+\mathbf{w}_{a, t} . \tag{4.10}
\end{equation*}
$$

Note this is a simplified model compared to Equation (3.9). Bias effects are neglected and acceleration itself is modeled as a simple random process. Since this filter is only aimed at improving orientation estimates, this simple model suffices.

## Inclination estimation

Based on the sensor model, the inclination estimation process for both the gyroscope and accelerometer systems can be described. Figure 4.5 shows the estimation process, based on the sensor model previously described. The acceleration ${ }^{G} \hat{\mathbf{a}}_{t}^{-}$, gyroscope bias $\hat{\mathbf{b}}_{t}^{-}$and angular velocity $\hat{\omega}_{t}^{-}$are calculated using equations 4.10, 3.6 and 3.4 respectively while setting the noise factors to zero. The orientation estimation ${ }^{G S} \hat{\mathbf{R}}_{t}^{-}$is calculated by strapdown integration of the previous orientation estimate ${ }^{G S} \hat{\mathbf{R}}_{t-1}^{+}$, together with the current angular velocity $\hat{\omega}_{t}^{-}$. Inclination estimation ${ }^{S} \hat{\mathbf{Z}}_{G, t}^{-}$can then be extracted from the orientation matrix. For the accelerometer system, the acceleration estimate is calculated in the sensor frame ${ }^{S} \hat{\mathbf{a}}_{G, t}^{-}$and subtracted from the sensor signal $\mathbf{y}_{A, t}$ to receive the gravity vector (according to Equation (3.9)). The gravity vector is then normalized and negated to produce the estimation of inclination ${ }^{S} \hat{\mathbf{Z}}_{A, t}^{-}$.

$$
\begin{equation*}
{ }^{S} \hat{\mathbf{Z}}_{A, t}^{-}=\frac{\mathbf{y}_{A, t}-{ }^{S} \hat{\mathbf{a}}_{t}}{\left|\mathbf{y}_{A, t}-{ }^{\hat{\mathbf{a}}_{t}}\right|} . \tag{4.11}
\end{equation*}
$$

## Error model

A complementary kalman filter uses a state space model representation to model the relationship between the model state variables $\mathbf{X}_{\varepsilon, t}$ (the error sources) and the inclination error predicted by the model [69].

$$
\begin{align*}
& \mathbf{X}_{\varepsilon, t}=\mathbf{A} \cdot \mathbf{x}_{\varepsilon, t-1}+\mathbf{w}_{t} \\
& \mathbf{Z}_{\varepsilon, t}=\mathbf{H} \cdot \mathbf{x}_{\varepsilon, t}+\mathbf{v}_{t} . \tag{4.12}
\end{align*}
$$



Figure 4.5: Inclination estimation process. The angular velocity $\hat{\omega}_{t}^{-}$is calculated from the gyroscope signal $\mathbf{y}_{G, t}$ minus the estimated bias. Strapdown integration is then used to produce the orientation estimate ${ }^{G S} \hat{\mathbf{R}}_{t}^{-}$and inclination ${ }^{G S} \hat{\mathbf{Z}}_{G, t}^{-}$. Acceleration ${ }^{G} \hat{\mathbf{a}}_{G, t}^{-}$is calculated from the previous estimation multiplied by a factor, and then rotated to the sensor frame. Finally, the inclination estimation ${ }^{S} \hat{\mathbf{Z}}_{A, t}^{-}$is calculated from the accelerometer signal and accelerometer estimation using 4.11

Where $\mathbf{X}_{\varepsilon, t}$ is the state, $\mathbf{A}$ is the state transition matrix, $\mathbf{Z}_{\varepsilon, t}$ is the measurement vector, and $\mathbf{H}$ is the observation model matrix (maps the state to the measurements). Both model and measurements exhibit uncertainties which are modeled as white noise processes $\mathbf{w}_{t}$ and $\mathbf{v}_{t}$ respectively, with covariances $\mathbf{Q}_{w, t}$ and $\mathbf{Q}_{v, t}$.

The measurement vector is the difference between the inclination estimations as shown in Figure 4.3

$$
\mathbf{Z}_{\varepsilon, t}={ }^{S} \hat{\mathbf{Z}}_{A}^{-}-{ }^{S} \hat{\mathbf{Z}}_{G}^{-}
$$

The difference in estimations is caused by several error factors. The major error sources compose the error state vector $\mathbf{X}_{\varepsilon, t}$ which the kalman filter tries to predict. Though real world system might be affected by a large number of error factors, it is sufficient to explicitly model only the major ones and represent all others as part of the model uncertainty matrix. For the inclination difference, the major error source is the accumulating orientation error $\theta_{\varepsilon}$ since inclination estimate is calculated by integrating the gyroscope signal with the estimate from the previous time step. The second error factor, the gyroscope bias error $\mathbf{b}_{\varepsilon}$, is relatively small but has immense effect over a period of time due to repeated integration [26].

$$
\mathbf{X}_{\varepsilon, t}=\left[\begin{array}{ll}
\theta_{\varepsilon, t} & \mathbf{b}_{\varepsilon, t}
\end{array}\right] .
$$

Error propagation The bias prediction error can be found by using the bias model 3.6 and the bias prediction equations.

$$
\begin{align*}
\mathbf{b}_{\varepsilon, t}^{-} & =\hat{\mathbf{b}}_{t}^{-}-\mathbf{b}_{t} \\
& =\hat{\mathbf{b}}_{t-1}^{-}-\mathbf{b}_{t-1}-\mathbf{w}_{b, t} \\
& =\mathbf{b}_{\varepsilon, t-1}^{-}-\mathbf{w}_{b, t} . \tag{4.13}
\end{align*}
$$

The orientation error propagation is given by

$$
\begin{equation*}
\theta_{\varepsilon, t}^{-}=\theta_{\varepsilon, t-1}^{-}-\mathbf{T} \mathbf{b}_{\varepsilon, t-1}^{-}+\mathbf{T} \mathbf{v}_{G, t} \tag{4.14}
\end{equation*}
$$

Using these results 4.13 and 4.14 , the state transition equations can be written in matrix form:

$$
\begin{align*}
\binom{\theta_{\varepsilon, t}}{\mathbf{b}_{\varepsilon, t}} & =\left[\begin{array}{rr}
1 & -T \\
0 & 1
\end{array}\right] \cdot\binom{\theta_{\varepsilon, t-1}}{\mathbf{b}_{\varepsilon, t-1}}+\binom{\mathbf{T} \mathbf{v}_{G, t}}{-\mathbf{w}_{b, t}} \\
\mathbf{X}_{\varepsilon, t} & =\left[\begin{array}{rr}
1 & -T \\
0 & 1
\end{array}\right] \cdot \mathbf{X}_{\varepsilon, t-1}+\binom{\mathbf{T}_{G, t}}{-\mathbf{w}_{b, t}} \tag{4.15}
\end{align*}
$$

Error covariance matrix $\mathbf{Q}_{w, t}$ can be easily produced. The noise processes are independent, so the covariance matrix becomes a simple diagonal matrix.

$$
\mathbf{Q}_{w, t}=\left[\begin{array}{cc}
T^{2} \mathbf{Q}_{v G} & 0  \tag{4.16}\\
0 & \mathbf{Q}_{b g}
\end{array}\right]
$$

Where $\mathbf{Q}_{v G}$ is the gyroscope noise covariance matrix and $\mathbf{Q}_{b g}$ is the very small covariance matrix of the bias noise.

Relationship between filter inputs and error states The error of the gyroscope based inclination estimate can be obtained from the formula [60]

$$
\begin{equation*}
{ }^{G S} \hat{R}={ }^{G S} R \cdot\left(I+\left[\theta_{\varepsilon} \times\right]\right. \tag{4.17}
\end{equation*}
$$

Since the inclination ${ }^{S} \hat{\mathbf{Z}}_{G}^{-}$is simply one of the rows of the rotation matrix, the same relationship holds (for small errors)

$$
\begin{equation*}
{ }^{S} \hat{\mathbf{Z}}_{G}^{-}={ }^{S} \hat{\mathbf{Z}}_{G}^{-} \cdot\left(I+\left[\theta_{\varepsilon} \times\right]\right) \tag{4.18}
\end{equation*}
$$

The accelerometer-based inclination estimate ${ }^{S} \hat{\mathbf{Z}}_{A}^{-}$depends on the error in estimated acceleration and the accelerometer sensor noise. Since the acceleromter is expressed in the sensor co-ordinate system, orientation error is also a possible error factor. The error in predicted acceleration in the global co-ordinate system can be found by using Equation (4.10)

$$
\begin{align*}
{ }^{G} \mathbf{a}_{\varepsilon, t}^{-} & ={ }^{G} \hat{\mathbf{a}}_{t}^{-}-{ }^{G} \mathbf{a}_{t} \\
& =c_{a} \cdot{ }^{G} \hat{\mathbf{a}}_{t-1}^{-}-\left(c_{a} \cdot{ }^{G} \mathbf{a}_{t-1}+\mathbf{w}_{a, t}\right) \\
& =c_{a} \cdot{ }^{G} \mathbf{a}_{\varepsilon, t-1}-\mathbf{w}_{a, t} . \tag{4.19}
\end{align*}
$$

In order to calculate ${ }^{S} \mathbf{a}_{\varepsilon, t}^{-}$in the sensor frame, the Equation (4.17) is used to compensate for orientation errors

$$
\begin{equation*}
{ }^{S} \mathbf{a}_{\varepsilon, t}^{-}=c_{a} \cdot{ }^{S} \mathbf{a}_{\varepsilon, t-1}-{ }^{S} \mathbf{w}_{a, t}+{ }^{S} \hat{\mathbf{a}}_{t}^{-} \times \theta_{\varepsilon, t} . \tag{4.20}
\end{equation*}
$$

To obtain the accelerometer-based inclination estimate, Equation (3.9) is used together with Equation (4.20)

$$
\begin{aligned}
\mathbf{y}_{A, t}-{ }^{S} \hat{\mathbf{a}}_{t} & ={ }^{S} \mathbf{a}_{t}-{ }^{S} \mathbf{g}_{t}+\mathbf{v}_{A, t}-{ }^{S} \hat{\mathbf{a}}_{t} \\
& ={ }^{S} \mathbf{a}_{\varepsilon, t}-{ }^{S} \mathbf{g}_{t}+\mathbf{v}_{A, t} \\
& =c_{a} \cdot{ }^{S} \mathbf{a}_{\varepsilon, t-1}-{ }^{S} \mathbf{w}_{a, t}+{ }^{S} \hat{\mathbf{a}}_{t}^{-} \times \theta_{\varepsilon, t}-{ }^{S} \mathbf{g}_{t}+\mathbf{v}_{A, t} .
\end{aligned}
$$

Putting it all together and using Equation (4.11) gives the following result

$$
\begin{equation*}
{ }^{S} \hat{\mathbf{Z}}_{A, t}^{-}=\mathbf{Z}_{t}+\frac{1}{g}\left(c_{a} \cdot{ }^{S} \mathbf{a}_{\varepsilon, t-1}+{ }^{S} \hat{\mathbf{a}}_{t}^{-} \times \theta_{\varepsilon, t}-{ }^{S} \mathbf{w}_{a, t}+\mathbf{v}_{A, t}\right) \tag{4.21}
\end{equation*}
$$

Where the magnitude of acceleration is approximated by $g$ the gravitational force.

The difference between inclination estimates can be expressed using the error state
variables using Equations 4.11 and 4.18.

$$
\begin{aligned}
{ }^{S} \mathbf{Z}_{\varepsilon, t} & ={ }^{S} \hat{\mathbf{Z}}_{A, t}^{-}-{ }^{S} \hat{\mathbf{Z}}_{G, t}^{-} \\
& =\left({ }^{S} \hat{\mathbf{Z}}_{t}-\frac{{ }^{S} \hat{\mathbf{a}}_{t}^{-}}{g}\right) \times \theta_{\varepsilon, t}+\frac{1}{g}\left(c_{a} \cdot{ }^{S} \mathbf{a}_{\varepsilon, t-1}-{ }^{S} \mathbf{w}_{a, t}+\mathbf{v}_{A, t}\right) \\
& =\left(\left[\left({ }^{S} \hat{\mathbf{Z}}_{t}-\frac{{ }^{S} \hat{\mathbf{a}}_{t}^{-}}{g}\right) \times\right]\left[\begin{array}{c}
\cdots \\
0 \\
\cdots
\end{array}\right]\right) \cdot\binom{\theta_{\varepsilon, t}}{\mathbf{b}_{\varepsilon, t}}+\mathbf{v}_{t} .
\end{aligned}
$$

Where the noise term $\mathbf{v}_{t}$ is described by

$$
\begin{equation*}
\mathbf{v}_{t}=\frac{1}{g}\left(c_{a} \cdot{ }^{S} \mathbf{a}_{\varepsilon, t-1}-{ }^{S} \mathbf{w}_{a, t}+\mathbf{v}_{A, t}\right) \tag{4.22}
\end{equation*}
$$

The matrix $\mathbf{H}$ is a $3 \times 6$ matrix

$$
\mathbf{H}=\left(\left[\left({ }^{S} \hat{\mathbf{Z}}_{t}-\frac{S_{\hat{\mathbf{a}}_{t}^{-}}}{g}\right) \times\right]\left[\begin{array}{lll}
0 & 0 & 0  \tag{4.23}\\
0 & 0 & 0 \\
0 & 0 & 0
\end{array}\right]\right)
$$

Finally, the covariance matrix of the measurement noise can be derived from Equation (4.22)

$$
\mathbf{Q}_{v, t}=\frac{1}{g^{2}}\left(c_{a}^{2} \cdot \mathbf{Q}_{a, t-1}^{+}+\mathbf{Q}_{w a}+\mathbf{Q}_{v a}\right) .
$$

Where $\mathbf{Q}_{a, t-1}^{+}$is the aposteriori acceleration error covariance matrix. $\mathbf{Q}_{w a}$ is the covariance matrix of $\mathbf{w}_{a, t}$ and $\mathbf{Q}_{v a}$ is the covariance matrix of $\mathbf{v}_{A, t}$.

### 4.2 Location estimation using RSSI

RSSI-based tracking systems use Received Signal Strength Indication (RSSI) to get an estimate of the distance between transmitter and receiver (ranging) [49]. Estimations rely on a network of static anchor nodes, which measure the RSSI from the moving object. The main challenge is inferring the distance from the signal strength. It is not easy to model the radio propagation in the indoor environment because of severe multipath, low probability for availability of line-of-sight (LOS) path, and specific site parameters such as floor layout, moving objects, and numerous reflecting surfaces [70]. Using distance estimations from 2 or more anchor nodes (for 2D), the location can be
determined using triangulation or scene analysis. Triangulation (also known as rangebased RSSI) directly estimates the distance from the signal strength measurements and combines them to compute the object location. This method is more straight forward but is prone to large errors due to shadowing effects. On the other hand, scene analysis refers to algorithms that first collect features (fingerprints) of a scene and then estimate the location of an object by matching online measurements with the closest a priori location fingerprints [48]. The fingerprinting algorithms have usually two stages, an offline calibration stage and the online stage. During the offline stage, a site survey is performed. The location coordinates and respective signal strengths from the anchor nodes are collected. During the online stage, the currently observed signal strengths are used to locate the object according to the previously recorded map.

The RSSI-based algorithm used in this research is a described in detail in previous work by Blumrozen et al. [16]. The algorithm uses an innovative fingerprinting method to tune the channel model parameters. During the offline stage, the optimal transmission power is determined. Maximizing the dynamic range of the RSSI measurements can lead to significant improvement in estimation accuracy [71]. Insufficient transmission power may lead to high packet loss while high transmission power can lead to saturation of the RSSI measurements and distort the distance estimation. The dynamic range is adaptively determined using a new method described in [16]. The second part of the offline stage is the calibration process which is based on log fitting of the RSSI measurements and approximating the power offset and the channel exponent.

During the online stage, RSSI measurements are collected and preprocessed. First, measurements from the various nodes are synchronized. Second, the range is estimated using the Equation (3.14)

$$
\begin{equation*}
L r_{i}^{t}=L t+A-q 10 \log _{10} d_{i}^{t}+\alpha^{t} \tag{4.24}
\end{equation*}
$$

Finally, the location is estimated using trilateration, based on a method described in [72]. Each range estimation forms a circle around the anchor node, these circles have two intersection points. Choosing the closest point to the object location satisfies the Maximum A Posteriori criterion. This is illustrated in Figure 4.6.


Figure 4.6: Selecting the intersection point closest to the object location

### 4.3 Location estimation using sensor fusion

This section describes the design of an optimum filter for location estimation using sensor data fusion as presented in Figure 4.7. The filter uses inertial sensors data and estimates location using strap down integration as outlined in Section 4.1. RSSI data is used as a second source of location information and an estimate is calculated as detailed in Section 4.2. The process of optimally fusing the two location estimations is performed using a kalman fitler.

A complementary kalman filter is designed to estimate positioning, velocity and orientation errors. As illustrated in Figure 4.7 the filter uses two independent estimations of location, each with different accuracies and error sources. The positioning difference $\hat{\mathbf{p}}_{I N S}^{-}-\hat{\mathbf{p}}_{R S S I}^{-}$is modeled as a function of errors in both measurement systems, particulary location, velocity and orientation errors in the gyroscope system. The system uses a feedback design to constantly fix the inertial system estimations according to the kalman filter corrections. Future work will also include feedback to the RSSI system, enabling online-calibration.

## Error model

As explained above in 4.1.2, a complementary kalman filter uses a state space model representation to model the relationship between the model state variables $\mathbf{X}_{\varepsilon, t}$ (the error sources) and the error predicted by the model. The general state space model


Figure 4.7: Top level design of the system. The system is composed of two independent sub systems that estimate the location, the kalman filter estimates the errors and corrects them using a feedback loop. Two estimations of the location are used, $\hat{\mathbf{p}}_{I N S}^{-}$and $\hat{\mathbf{p}}_{R S S I}^{-}$. The difference between them is a function of the location, velocity, orientation and bias errors ( $\left.\hat{p}_{\varepsilon}, \hat{v}_{\varepsilon}, \hat{\theta}_{\varepsilon}, \hat{\mathbf{b}}_{a, \varepsilon}, \hat{\mathbf{b}}_{g, \varepsilon}\right)$ which the kalman filter estimates. The uncertainties of the measurements and model are expressed in terms of covariances, $\mathbf{Q}_{w a}$ and $\mathbf{Q}_{w g}$ for bias uncertainties, $\mathbf{Q}_{v A}$ and $\mathbf{Q}_{v G}$ for sensor white noise , $\mathbf{Q}_{v R}$ for RSSI estimation uncertainties and $\mathbf{Q}_{w r}$ for RSSI shadowing uncertainties
representation was described in Equation (4.12).
The measurement vector is the difference between the location estimations as depicted in Figure 4.7

$$
\begin{equation*}
\mathbf{Z}_{\varepsilon, t}=\hat{\mathbf{p}}_{i n}^{-}-\hat{\mathbf{p}}_{r s s i}^{-} . \tag{4.25}
\end{equation*}
$$

The difference in location estimations is modeled as a function of several error sources. These errors compose the error state vector $\mathbf{X}_{\varepsilon, t}$ which the kalman filter tries to predict. For the location difference, the dominant error factors are the notorious inertial navigation systems caveats - integration drift and sensor bias [26]. The inertial system estimates location by integrating orientation, velocity and location, each integral is a potential error source due to integration $\operatorname{drift}\left(\hat{\theta}_{\varepsilon}, \hat{v}_{\varepsilon}, \hat{p}_{\varepsilon}\right.$ respectively). The second factor to consider is sensor bias error. Although the bias offset is very small, if not mitigated it might cause large errors due to repeated integration of error each time step [26]. $\hat{\mathbf{b}}_{a, \varepsilon}$ and $\hat{\mathbf{b}}_{g, \varepsilon}$ represent the accelerometer bias error and gyroscope bias
error, respectively.

$$
\begin{equation*}
\mathbf{X}_{\varepsilon, t}=\left[\hat{p}_{\varepsilon}, \hat{v}_{\varepsilon}, \hat{\theta}_{\varepsilon}, \hat{\mathbf{b}}_{a, \varepsilon}, \hat{\mathbf{b}}_{g, \varepsilon}\right] . \tag{4.26}
\end{equation*}
$$

The error state variables that compose $\mathbf{X}_{\varepsilon, t}$ are defined as follows. $\hat{p}_{\varepsilon}$ is the difference in location between the real location and estimated location by the inertial system

$$
\begin{equation*}
\hat{p}_{\varepsilon}=p_{i n, t}-p_{t} \tag{4.27}
\end{equation*}
$$

Similarly, the variable $\hat{v}_{\varepsilon}$ is the difference between velocity estimations

$$
\begin{equation*}
\hat{v}_{\varepsilon}=v_{i n, t}-v_{t} \tag{4.28}
\end{equation*}
$$

The orientation difference $\hat{\theta}_{\varepsilon}$ is defined to be the rotation vector between the estimated rotation and the real rotation. Recall for small rotations, the relation between orientations is given by Equation (3.3). Hence, the following formula can be derived

$$
\begin{equation*}
R_{i n, t}^{G S}=R_{t}^{G S}\left(I+\left[\hat{\theta}_{\varepsilon} \times\right]\right) \tag{4.29}
\end{equation*}
$$

Another form of this property can be written in terms of rotation vectors, where $\theta_{i n, t}, \theta_{t}$ are the rotation vectors equivalent to $R_{i n, t}^{G S}, R_{t}^{G S}$ respectively.

$$
\begin{equation*}
\hat{\theta}_{\varepsilon}=\theta_{i n, t}-\theta_{t} \tag{4.30}
\end{equation*}
$$

Finally, the state variables corresponding to bias errors $\hat{\mathbf{b}}_{a, \varepsilon}, \hat{\mathbf{b}}_{g, \varepsilon}$ are simply defined as

$$
\begin{align*}
& \hat{\mathbf{b}}_{a, \varepsilon}=\mathbf{b}_{a, i n s}-\mathbf{b}_{a} \\
& \hat{\mathbf{b}}_{g, \varepsilon}=\mathbf{b}_{g, \text { ins }}-\mathbf{b}_{g} \tag{4.31}
\end{align*}
$$

Error propagation The error propagation equations are derived in continuous time and are later transformed to discrete time. Using continuous time, the derivation is simpler and more intuitive since human motion is a physical continuous process. When using continuous terms, the error propagation equations express the derivative of the variables.

First, the variable $\hat{p}_{\varepsilon}$ is examined. By definition, it is simply the difference between the estimated and real location [see Equation (4.27)]. Calculating the derivative is
trivial and gives the following result

$$
\begin{align*}
\dot{p_{\varepsilon}} & =\dot{p}_{i n, t}-\dot{p}_{t} \\
& =\mathbf{v}_{i n, t}-\mathbf{v}_{t} \\
& =\hat{v}_{\varepsilon} . \tag{4.32}
\end{align*}
$$

Where the last result was derived according to Equation (4.28). The derivation for $\hat{v}_{\varepsilon}$ is similar

$$
\begin{align*}
\dot{v}_{\varepsilon} & =\dot{v}_{i n, t}-\dot{v}_{t} \\
& =\mathbf{a}_{i n, t}-\mathbf{a}_{t} \\
& =\hat{a}_{\varepsilon} . \tag{4.33}
\end{align*}
$$

Where $a_{\varepsilon}$ is defined to be the difference between acceleration estimations. This variable is not a state variable, as it is expressed by $\theta_{\varepsilon}$ as the following derivation shows. Recall from Equation (4.7) that

$$
\begin{equation*}
{ }^{G} \mathbf{a}_{i n, t}={ }^{G} \mathbf{y}_{A, t}-{ }^{G S} R_{i n, t}{ }^{S} \mathbf{b}_{i n, a t}+{ }^{G} \mathbf{g}_{i n, t} . \tag{4.34}
\end{equation*}
$$

Substituting ${ }^{S} \mathbf{y}_{A, t}$ according to Equation (3.9) yields

$$
{ }^{G} \mathbf{y}_{A, t}=R_{i n, t}^{G S} \cdot\left({ }^{S} \mathbf{a}_{t}-{ }^{S} \mathbf{g}_{t}+\mathbf{b}_{a, t}+\mathbf{v}_{A, t}\right) .
$$

Using Equation (4.29) for the variable $R_{i n, t}^{G S}$

$$
\begin{align*}
{ }^{G} \mathbf{y}_{A, t} & =R_{t}^{G S} \cdot\left(I+\left[\hat{\theta}_{\varepsilon} \times\right]\right) \cdot\left({ }^{S} \mathbf{a}_{t}-{ }^{S} \mathbf{g}_{t}\right)+R_{i n, t}^{G S} \cdot \mathbf{b}_{a, t}+R_{i n, t}^{G S} \cdot \mathbf{v}_{A, t} \\
& =R_{t}^{G S} \cdot{ }^{S} \mathbf{a}_{t}-R_{t}^{G S} \cdot{ }^{S} \mathbf{g}_{t}+\hat{\theta}_{\varepsilon} \times{ }^{S} \mathbf{a}_{t}-\hat{\theta}_{\varepsilon} \times{ }^{S} \mathbf{g}_{t}+R_{i n, t}^{G S} \cdot \mathbf{b}_{a, t}+R_{i n, t}^{G S} \cdot \mathbf{v}_{A, t} \\
& ={ }^{G} \mathbf{a}_{t}-{ }^{G} \mathbf{g}_{t}+\hat{\theta}_{\varepsilon} \times{ }^{S} \mathbf{a}_{t}-\hat{\theta}_{\varepsilon} \times{ }^{S} \mathbf{g}_{t}+R_{i n, t}^{G S} \cdot \mathbf{b}_{a, t}+R_{i n, t}^{G S} \cdot \mathbf{v}_{A, t} . \tag{4.35}
\end{align*}
$$

Returning to the Equation (4.33) and substituting ${ }^{G} \mathbf{a}_{i n, t}$ according to Equation (4.34) yields

$$
\hat{a}_{\varepsilon}={ }^{G} \mathbf{y}_{A, t}-{ }^{G S} R_{i n, t}{ }^{S} \mathbf{b}_{i n, a t}+{ }^{G} \mathbf{g}_{i n, t}-\mathbf{a}_{t} .
$$

Using the result 4.35 from above, the following can be derived

$$
\begin{align*}
\hat{a}_{\varepsilon} & =\hat{\theta}_{\varepsilon} \times{ }^{S} \mathbf{a}_{t}-\hat{\theta}_{\varepsilon} \times{ }^{S} \mathbf{g}_{t}+R_{i n, t}^{G S} \cdot\left(\mathbf{b}_{a, t}-{ }^{S} \mathbf{b}_{i n, a t}\right)+R_{i n, t}^{G S} \cdot \mathbf{v}_{A, t} \\
& =\hat{\theta}_{\varepsilon}\left(\left[{ }^{S} \mathbf{a}_{t} \times\right]-\left[{ }^{S} \mathbf{g}_{t} \times\right]\right)-R_{i n, t}^{G S} \cdot \mathbf{b}_{a, \varepsilon}+R_{i n, t}^{G S} \cdot \mathbf{v}_{A, t} . \tag{4.36}
\end{align*}
$$

Where the last transition was according to the definition of $\mathbf{b}_{a, \varepsilon}$ in Equation (4.31). For the variable $\theta_{\varepsilon}$, using Equation (4.30) yields

$$
\begin{equation*}
\dot{\theta}_{\varepsilon}=\theta_{i n, t}-\dot{\theta}_{t} . \tag{4.37}
\end{equation*}
$$

Recall Equations 4.3 and 4.5 for $\theta_{i n, t}$ and $\dot{\theta}_{t}$

$$
\begin{aligned}
\theta_{i n, t} & =\mathbf{y}_{G, t}-\mathbf{b}_{i n, g t} \\
\dot{\theta_{t}} & =\omega_{t} .
\end{aligned}
$$

Substituting these properties to Equation (4.37) yields

$$
\dot{\theta}_{\varepsilon}=\mathbf{y}_{G, t}-\mathbf{b}_{i n, g t}-\omega_{t} .
$$

$\mathbf{y}_{G, t}$ can be replaced according to Equation (3.4)

$$
\begin{align*}
\dot{\theta}_{\varepsilon} & =\omega_{t}+\mathbf{b}_{g t}+\mathbf{v}_{G, t}-\mathbf{b}_{i n s, g t}-\omega_{t} \\
& =-\mathbf{b}_{\varepsilon}+\mathbf{v}_{G, t} . \tag{4.38}
\end{align*}
$$

Where the last result is derived using 4.31 Finally, for the bias error derivative, recall Equations 3.13 and 3.8

$$
\begin{align*}
\dot{\mathbf{b}}_{a, t} & =\mathbf{w}_{b a, t} \\
\dot{\mathbf{b}}_{g, t} & =\mathbf{w}_{b g, t} . \tag{4.39}
\end{align*}
$$

Using also Equations 4.1 and 4.6 we get

$$
\begin{aligned}
& \dot{\mathbf{b}}_{i n, a t}=0 \\
& \dot{\mathbf{b}}_{i n, g t}=0 .
\end{aligned}
$$

Where the derivative is zero since the estimation is a constant. Substituting these
expressions in the bias error definition, yields -

$$
\begin{aligned}
\dot{\mathbf{b}}_{a, \varepsilon} & =\mathbf{b}_{a, i n s}-\dot{\mathbf{b}_{a}} \\
& =-\mathbf{w}_{b a, t} \\
\dot{\mathbf{b}}_{g, \varepsilon} & =\mathbf{b}_{g, i n s}-\dot{\mathbf{b}_{g}} \\
& =-\mathbf{w}_{b g, t} .
\end{aligned}
$$

Let us summarize the linear differential equations obtained so far,

$$
\begin{aligned}
\dot{p}_{\varepsilon} & =v_{\varepsilon} \\
\dot{v}_{\varepsilon} & =\hat{\theta}_{\varepsilon}\left(\left[{ }^{S} \mathbf{a}_{t} \times\right]-\left[{ }^{S} \mathbf{g}_{t} \times\right]\right)-R_{i n, t}^{G S} \cdot \mathbf{b}_{a, \varepsilon}+R_{i n, t}^{G S} \cdot \mathbf{v}_{A, t} \\
\dot{\theta}_{\varepsilon} & =-\mathbf{b}_{\varepsilon}+\mathbf{v}_{G, t} \\
\dot{\mathbf{b}}_{a, t} & =-\mathbf{w}_{b a, t} \\
\dot{\mathbf{b}}_{g, t} & =-\mathbf{w}_{b g, t} .
\end{aligned}
$$

According to Equations 4.32,4.36, 4.39,4.38 for location,velocity,bias and orientation errors respectively. Writing these equations in matrix representation, yields

$$
\left[\begin{array}{c}
\dot{p}_{\varepsilon}  \tag{4.40}\\
\dot{v}_{\varepsilon} \\
\dot{\theta}_{\varepsilon} \\
\dot{\mathbf{b}}_{g, t} \\
\dot{\mathbf{b}}_{a, t}
\end{array}\right]=\mathbf{A}_{\text {cont }}\left[\begin{array}{c}
p_{\varepsilon} \\
v_{\varepsilon} \\
\theta_{\varepsilon} \\
\mathbf{b}_{g, t} \\
\mathbf{b}_{a, t}
\end{array}\right]+\left[\begin{array}{c}
0 \\
\mathbf{v}_{A, t} \\
\mathbf{v}_{G, t} \\
-\mathbf{w}_{b g, t} \\
-\mathbf{w}_{b a, t}
\end{array}\right] .
$$

Where all variables are three dimensional vectors and $\mathbf{A}_{\text {cont }}$ is a $15 \times 15$ matrix.

$$
\mathbf{A}_{\text {cont }}=\left[\begin{array}{ccccc}
\mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0}  \tag{4.41}\\
\mathbf{0} & \mathbf{0} & \left(\left[{ }^{S} \mathbf{a}_{t} \times\right]-\left[{ }^{S} \mathbf{g}_{t} \times\right]\right) & \mathbf{0} & -R_{\text {in,t }}^{G S} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{I} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}
\end{array}\right]
$$

Where $\mathbf{0}$ denotes a $3 \times 3$ zero matrix and $\mathbf{I}$ is a $3 \times 3$ unity matrix.
Since the sensors sample the process in discrete times, we shall transform the
equations to discrete-time dynamics. Recall that [66] for a continuous time linear system given by the dynamics

$$
\begin{aligned}
\dot{x} & =B x \\
y & =C x
\end{aligned}
$$

The transformation to discrete time dynamics is given by

$$
\begin{align*}
\mathbf{x}_{t} & =e^{\mathbf{B} \Delta t} \mathbf{x}_{t-1} \\
\mathbf{y}_{t} & =\mathbf{C} \mathbf{x}_{t} \tag{4.42}
\end{align*}
$$

Where $e^{\mathbf{A}_{\text {cont }} \Delta t}$ can be calculated using the formula [73]

$$
e^{\mathbf{B} t}=\sum_{j=0}^{\infty} \frac{(\mathbf{B} t)^{j}}{j!}
$$

For the state transition matrix $\mathbf{A}_{\text {cont }}$ described in 4.41, the summation terms $\mathbf{A}_{\text {cont }}^{i}$ can be easily computed as follows

$$
\begin{aligned}
& \mathbf{A}_{\text {cont }}^{0}=I \\
& \mathbf{A}_{\text {cont }}^{2}=\left[\begin{array}{llllc}
0^{3} & \mathbf{0} & \left(\left[{ }^{S} \mathbf{a}_{t} \times\right]-\left[{ }^{S} \mathbf{g}_{t} \times\right]\right) & \mathbf{0} & -R_{i n, t}^{G S} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & -\left(\left[{ }^{S} \mathbf{a}_{t} \times\right]-\left[{ }^{S} \mathbf{g}_{t} \times\right]\right) & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}
\end{array}\right] \\
& \mathbf{A}_{\text {cont }}^{3}=\left[\begin{array}{lllcc}
\mathbf{0} & \mathbf{0} & \mathbf{0} & -\left(\left[{ }^{S} \mathbf{a}_{t} \times\right]-\left[{ }^{S} \mathbf{g}_{t} \times\right]\right) & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}
\end{array}\right] \\
& \mathbf{A}_{\text {cont }}^{4}=0 .
\end{aligned}
$$

Thus, the formula for $e^{\mathbf{A}_{\text {cont }} \Delta t}$ is given by

$$
\begin{align*}
e^{\mathbf{A}_{\text {cont }} \Delta t} & =\mathbf{A}_{\text {cont }}^{0}+\mathbf{A}_{\text {cont }}^{1} \Delta t+\mathbf{A}_{\text {cont }}^{2} \frac{\Delta t^{2}}{2}+\mathbf{A}_{\text {cont }}^{3} \frac{\Delta t^{3}}{6} \\
& {\left[\begin{array}{cccccc}
\mathbf{I} & \Delta t \cdot \mathbf{I} & \frac{\Delta t^{2}}{2}\left(\left[{ }^{S} \mathbf{a}_{t} \times\right]-\left[{ }^{S} \mathbf{g}_{t} \times\right]\right) & -\frac{\Delta t^{3}}{6}\left(\left[{ }^{S} \mathbf{a}_{t} \times\right]-\left[{ }^{S} \mathbf{g}_{t} \times\right]\right) & -\frac{\Delta t^{2}}{2} R_{i n, t}^{G S} \\
\mathbf{0} & \mathbf{I} & \Delta t\left(\left[{ }^{S} \mathbf{a}_{t} \times\right]-\left[{ }^{S} \mathbf{g}_{t} \times\right]\right) & -\frac{\Delta t^{2}}{2}\left(\left[{ }^{S} \mathbf{a}_{t} \times\right]-\left[{ }^{S} \mathbf{g}_{t} \times\right]\right) & -\Delta t R_{i n, t}^{G S} \\
\mathbf{0} & \mathbf{0} & \mathbf{I} & \Delta t^{3} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}
\end{array}\right] . } \tag{4.43}
\end{align*}
$$

Using the discrete time transformation Eq. 4.42 with the continuous time dynamics system Eq.4.40 gives us the following discrete time dynamics system for error propagation -

$$
\left[\begin{array}{c}
p_{\varepsilon, t}  \tag{4.44}\\
v_{\varepsilon, t} \\
\theta_{\varepsilon, t} \\
\mathbf{b}_{g, t} \\
\mathbf{b}_{a, t}
\end{array}\right]=\mathbf{A}\left[\begin{array}{c}
p_{\varepsilon, t-1} \\
v_{\varepsilon, t-1} \\
\theta_{\varepsilon, t-1} \\
\mathbf{b}_{g, t-1} \\
\mathbf{b}_{a, t-1}
\end{array}\right]+\left[\begin{array}{c}
0 \\
\mathbf{v}_{A, t} \\
\mathbf{v}_{G, t} \\
-\mathbf{w}_{b g, t} \\
-\mathbf{w}_{b a, t}
\end{array}\right] \Delta t
$$

Where $A$ is an $15 \times 15$ matrix as described in Equation (4.43). Note that the matrix $A$ depends on current acceleration and orientation of the object, thus it is recomputed each cycle of the filter.

The model noise covariance matrix can be easily computed, since it is composed of independent white noise processes.

$$
\mathbf{Q}_{w, t}=\left[\begin{array}{ccccc}
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}  \tag{4.45}\\
\mathbf{0} & \Delta t^{2} \mathbf{Q}_{v A} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \Delta t^{2} \mathbf{Q}_{v G} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \Delta t^{2} \mathbf{Q}_{w g} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \Delta t^{2} \mathbf{Q}_{w a}
\end{array}\right]
$$

where $\mathbf{Q}_{v G}, \mathbf{Q}_{v A}, \mathbf{Q}_{w g}, \mathbf{Q}_{w a}$ are white noise process covariance matrices as defined in Eq. (3.5),Eq.3.10,Eq.3.7,Eq.3.12 respectively.

Relationship between filter input and error states The error of the RSSI based location estimate is given by Equation (3.15)

$$
\begin{equation*}
\hat{\mathbf{p}}_{r s s i, t}=\mathbf{p}_{t}+\mathbf{b}_{r, t}+\mathbf{v}_{R, t} . \tag{4.46}
\end{equation*}
$$

Where $\mathbf{p}_{t}$ is the real location, $\mathbf{b}_{r, t}$ is a correlated noise process and $\mathbf{v}_{R, t}$ a white noise gaussian process. The expression for the location estimation difference was found by substituting Eq.4.46 and Eq.4.27 into Eq.4.25

$$
\begin{aligned}
\mathbf{Z}_{\varepsilon, t} & =\hat{\mathbf{p}}_{i n}^{-}-\hat{\mathbf{p}}_{r s s i}^{-} \\
& =\left(\hat{\mathbf{p}}_{i n}^{-}-\mathbf{p}_{t}\right)-\left(\hat{\mathbf{p}}_{r s s i}^{-}-\mathbf{p}_{t}\right) \\
& =\hat{p}_{\varepsilon}-\left(\mathbf{b}_{r, t}+\mathbf{v}_{R, t}\right)
\end{aligned}
$$

Rewriting the last equation according to the kalman filter formulation, yields

$$
\mathbf{Z}_{\varepsilon, t}=\left[\begin{array}{lllll}
\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \tag{4.47}
\end{array}\right] \mathbf{X}_{\varepsilon, t}-\left(\mathbf{b}_{r, t}+\mathbf{v}_{R, t}\right)
$$

Where $\mathbf{I}$ is a $3 \times 3$ identity matrix, 0 is a $3 \times 3$ zeros matrix and $\mathbf{X}_{\varepsilon, t}$ is the error state vector defined in Equation (4.26).

Kalman filter can be derived to work with colored measurement noise process by augmenting the state vector with the noise process [67]. This derivation is needed since the measurement noise expression $\left(\mathbf{b}_{r, t}+\mathbf{v}_{R, t}\right)$ in Equation (4.47) contains correlated components and thus does not comply with the basic kalman filter assumptions of white noise processes only. Recall from Equation (3.17) that the correlated noise process model is

$$
\begin{equation*}
\mathbf{b}_{r, t}=e^{-\frac{|\Delta t|}{\tau}} \mathbf{b}_{r, t-1}+\mathbf{w}_{r, t} . \tag{4.48}
\end{equation*}
$$

Augmenting the state vector from Eq. (4.44) with the noise process given in Eq. (4.48) yields the state space representation of the kalman filter

$$
\mathbf{X}_{\varepsilon, t}^{\prime}=\mathbf{A}^{\prime} \mathbf{X}_{\varepsilon, t-1}^{\prime}+\left[\begin{array}{c}
0 \\
\mathbf{v}_{A, t} \\
\mathbf{v}_{G, t} \\
-\mathbf{w}_{b g, t} \\
-\mathbf{w}_{b a, t} \\
\mathbf{w}_{b r, t}
\end{array}\right] \Delta t
$$

Where $\mathbf{X}_{\varepsilon, t}^{\prime}$ is the augmented state vector

$$
\mathbf{X}_{\varepsilon, t}^{\prime}=\left[\begin{array}{c}
p_{\varepsilon, t} \\
v_{\varepsilon, t} \\
\theta_{\varepsilon, t} \\
\mathbf{b}_{g, t} \\
\mathbf{b}_{a, t} \\
\mathbf{b}_{r, t}
\end{array}\right] .
$$

$\mathbf{A}^{\prime}$ is the augmented state transition matrix of size $18 \times 18$ given by

$$
\mathbf{A}^{\prime}=\left[\begin{array}{cccccc}
\mathbf{I} & \Delta t \cdot \mathbf{I} & \frac{\Delta t^{2}}{2}\left(\left[{ }^{S} \mathbf{a}_{t} \times\right]-\left[{ }^{S} \mathbf{g}_{t} \times\right]\right) & -\frac{\Delta t^{3}}{6}\left(\left[{ }^{S} \mathbf{a}_{t} \times\right]-\left[{ }^{S} \mathbf{g}_{t} \times\right]\right) & -\frac{\Delta t^{2}}{2} R_{i n, t}^{G S} & \mathbf{0} \\
\mathbf{0} & \mathbf{I} & \Delta t\left(\left[{ }^{S} \mathbf{a}_{t} \times\right]-\left[{ }^{S} \mathbf{g}_{t} \times\right]\right) & -\frac{\Delta t^{2}}{2}\left(\left[{ }^{S} \mathbf{a}_{t} \times\right]-\left[{ }^{S} \mathbf{g}_{t} \times\right]\right) & -\Delta t R_{i n, t}^{G S} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{I} & \Delta t^{3} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & e^{-\frac{|\Delta t|}{\tau}} \mathbf{I}
\end{array}\right]
$$

The relationship between filter inputs and error states was found be rewriting Eq.4.47 with the augmented state vector

$$
\mathbf{Z}_{\varepsilon, t}=\left[\begin{array}{llllll}
\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{I}
\end{array}\right] \mathbf{X}_{\varepsilon, t}^{\prime}-\mathbf{v}_{R, t}
$$

The model noise covariance matrix was found by augmenting the covariance matrix 4.45 with the $\mathbf{w}_{b r, t}$ noise process which is independent of the other noise processes, yielding

$$
\mathbf{Q}_{w, t}=\left[\begin{array}{cccccc}
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \Delta t^{2} \mathbf{Q}_{v A} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \Delta t^{2} \mathbf{Q}_{v G} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \Delta t^{2} \mathbf{Q}_{w g} & \mathbf{0} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \Delta t^{2} \mathbf{Q}_{w a} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \Delta t^{2} \mathbf{Q}_{w r}
\end{array}\right]
$$

Where $\mathbf{Q}_{w r}$ is the covariance matrix of $\mathbf{w}_{b r, t}$ as defined in Eq. (3.18). The measurement noise covariance matrix is simply given by

$$
\mathbf{Q}_{v, t}=\left[\mathbf{Q}_{v R}\right] .
$$

Where $\mathbf{Q}_{v R}$ is the covariance matrix of $\mathbf{v}_{R, t}$ as defined in Eq. (3.16).

## Chapter 5

## Experimental methods

Experiments were conducted to verify the effectiveness and robustness of the tracking filter. The experiments were carried out on a $50 \times 50 \mathrm{~cm}$ surface in an indoor environment. The experiments aim was to track the position of a moving hand, along the surface of the table, as depicted in Figure 5.1.Data was collected in real-time and post-processed using a program written in Matlab. Sampling rate was approximately 50 Hz . The tracking results were tested by comparison of the location estimation to the location information that was obtained by a laboratory-bound 3D optical tracking system Polaris [74].

The experiment design is presented in Figure 5.2. The sensors data is acquired and calibrated as explained in Section 5.4. The data is then synchronized and interpolated to create a uniform time stamp and sampling rate, see Section 5.5 for more details. Finally, location is estimated as explained in 4. The results are compared to the reference tracking data provided by the polaris system.

### 5.1 Experiment setup

The experiment setup is depicted in Fig. 5.1. Inertial movement information was acquired using a SHIMMER sensor attached to the moving hand [75]. For RSSI measurements, two BSN sensor nodes [76] were used as anchor nodes, and the third was used as a mobile node. Location estimation was accomplished by fusion of both sensor systems.

The current implementation of the RSSI tracking system consists of only 2 anchor nodes, thus providing only 2D tracking capability. Hence, the experiments were limited to 2D movement across the experiment area. Intensive simulation testing was


Figure 5.1: Experiment setup. Tracking of the hand movement across the 2D surface is accomplished by attaching several sensors to the moving hand. For inertial movement information, the SHIMMER sensor is used. For location estimation using RSSI, two BSN nodes are used as anchor nodes and one BSN node is attached to the hand. The system results are compared to reference information provided by the optical tracking system Polaris. The Polaris system uses a passive marker mounted on the moving hand.
performed to further explore 3D movement scenarios.

### 5.1.1 Inertial system setup

The sensor platform used for inertial navigation data is the Intels Digital Health Groups platform for Sensing Health with Intelligence, Modularity, Mobility, and Experimental Re-usability, or SHIMMER. SHIMMER (Figure 5.4) consists of a TI MSP430 microprocessor; a Chipcon CC2420 IEEE 802.15.4 2.4 GHz radio; a MicroSD card slot; a triaxial MEMS accelerometer, and a Bluetooth radio which allows streaming of sensor data at high rates. Internal and external connectors allow new sensor boards to be interfaced to the device, expanding its capabilities. A triaxial gyroscope board using two InvenSense IDG-300 dual-axis gyroscope chips was designed for internal expansion. One of these gyroscopes is mounted perpendicularly to the main sensor board (see Figure 5.4).

The SHIMMER device combines computation, radio communication, high-fidelity


Figure 5.2: Block diagram of the experiment data flow. Sensor data from the SHIMMER and BSN node is calibrated and pre-processed. All data sources are synchronized, and a uniform time stamp is determined. The location estimation process is composed of the INS and RSSI location estimations ( $\hat{p}_{I N S}^{-}$and $\hat{p}_{R S S I}^{-}$respectively). The kalman filter uses both estimations to predict the errors $\hat{p}_{\varepsilon}$. The INS errors are corrected in a feedback loop and the location estimation is computed $\hat{p}_{I N S}^{+}$. Polaris location estimation $\hat{p}_{\text {polaris }}$ is then compared to results and the Mean Squared Error is calculated.
triaxial sensors, and a large flash memory into a tiny, wearable rugged plastic enclosure. It measures $1.75 \times 0.8 \times 0.5$ and weighs just 10 g .

### 5.1.2 RSSI system setup

The RSSI tracking was performed by two anchor nodes placed at $x$ and $y$ axes and a third node attached to the moving hand. The mobile node transmitted data packets. The anchor nodes received the packets and calculated the RSSI measurements.

The wireless nodes are composed of a BSN node [76] and a dipole antenna. A BSN node includes a processing unit (TI MSP430) and a wireless chip (Chipcon CC2420) [77]. The wireless transceiver has a built-in RSSI that provides a digital value in the range of -127 to 128 dBm . An omnidirectional antenna was added to each node to increase the transmission range. The antenna was made by bending and winding together a 10 cm wire and forming a dipole antenna of 5 cm , which is equivalent to half the length of the 802.15.4a wave length.


Figure 5.3: The mobile node attached to the hand. A SHIMMER sensor is used for capturing INS information, a BSN node transmits data packets for RSSI measurements and a Polaris passive marker is used by the Polaris system to track the moving hand


Figure 5.4: The SHIMMER wearable sensor platform. SHIMMER incorporates a TI MSP430 processor, a CC2420 IEEE 802.15.4 radio, a triaxial accelerometer, and a rechargeable Li-polymer battery. A triaxial gyroscope board is added as an internal expansion with two dual-axis gyroscope chips. The platform also includes a MicroSD slot supporting up to 2 GB of Flash memory

### 5.2 Comparison with optical tracking system

The polaris system is an optical measurement system that measures the 3D position and orientation of either passive or active markers [74]. The system is highly accurate, providing sub-millimeter accuracy for a limited range of operation. It is manufactured by Northern Digital Inc. (NDI) of Waterloo, Ontario, Canada and is the leading system used for Image Guided Surgery (IGS). The programming interface for the polaris system is provided by the IGSTK, a software toolkit for image-guided surgery applications [78].

The polaris system can instantaneously track up to 15 markers. The experiment setup uses two passive markers, as depicted in Figure 5.2. A static marker is placed in the origin of axis, position and orientation conforming with the co-ordinate system defined by the RSSI tracking system. The second marker is attached to the moving hand, tracking its position and orientation. The polaris system outputs the relative movement of the mobile marker in the co-ordinate system defined by the static marker.


Figure 5.5: The polaris tracking setup. Two passive markers are used. The first is a static marker, placed in the origin of axis defined by the RSSI tracking system. The second is a mobile marker attached to the moving hand.

### 5.3 Model parameters estimation

Before the Kalman filter was used, the model parameters were determined. The sensor noise variances $\mathbf{Q}_{v A}$ and $\mathbf{Q}_{v G}$ were found by taking the variance of the sensor signal
while the sensor was lying still on the laboratory floor. The bias instability parameters $\mathbf{Q}_{w a}$ and $\mathbf{Q}_{w g}$ were found using a technique known as Allan Variance [79].Allan Variance is a time domain analysis technique originally designed for characterizing noise and stability in clock systems. The technique can be applied to any signal to determine the character of the underlying noise processes.More specifically, the bias instability is the minimum value of the Allan deviation curve. For a full description of the Allan Variance technique see [80]. The noise parameter of the RSSI error model $\mathbf{Q}_{w r}$ was chosen to give reasonable results while the filter was tested

### 5.4 Sensor calibration

SHIMMER device calibration Shimmer devices use MEMs transducers for kinematic sensing. The current generation has a lower price point than previous generations but suffers from errors that may be insufficient for many applications [3]. Most prominent errors are offset errors that are be caused by offset variations from trim errors, mechanical stresses from the package and mounting, shifts due to temperature and due to aging. These variables can all change the offset. The other source of errors is scaling errors which affect the slope of the transfer function. The aim of the calibration process is to eliminate these errors.


Figure 5.6: Transfer function of the accelerometer sensor. $S$ is the slope of the transfer function. $V_{O F F}$ is the offset error.

The calibration process is performed for both accelerometers and gyroscopes at the beginning of each experiment session. Hence, it is required to be quick and
simple. The calibration scheme presented in this research is composed of one combined calibration sequence for calibrating all sensors (3 axis accelerometer, 3 axis gyroscope). The sequence is composed of steps, during each step the sensor is layed on one of its faces, every step is at least 2 seconds long. Changing from one position to the other, should be done by turning the sensor along one of its gyroscope axis. The calibration code automatically computes the steps and turns, so all that is required from the user is to traverse between all different faces and turns at least once, order doesn't matter.

Accelerometer calibration is simple, given two points on the transfer function (approximated as linear function), the slope and offset can be computed. As shown in Figure 5.7 the calibration code locates the steps and finds for each step the earthfacing sensor (only one axis measures $g$ or $-g$ at each time step). To reach an approximation for $-g$ all measurements from relevant steps are averaged ( $\hat{\mathbf{x}}_{\text {min }}^{a c c}$ ). The same process repeats for $+g\left(\hat{\mathbf{x}}_{\max }^{a c c}\right)$ for each axis. The two points on the slope are $\left(-g, \hat{\mathbf{x}}_{\text {min }}^{a c c}\right)$ and $\left(+g, \hat{\mathbf{x}}_{\text {max }}^{a c c}\right)$ where $g$ is the gravity force. The previously described process is not accurate since the measured surface is never exactly aligned to the ground, hence the measured force is not $g$, it is

$$
\begin{equation*}
g_{\text {surface }}=g \cos (\alpha) . \tag{5.1}
\end{equation*}
$$

Where $\alpha$ is the surface incline angle. The calibration process estimates the slope incline ( $\alpha$ ) by calculating the acceleration measured by the two other sensors. The following property holds (assuming $z$ is earth facing)

$$
\begin{equation*}
\sqrt{a_{x}^{2}+a_{y}^{2}}=g \sin (\alpha) \tag{5.2}
\end{equation*}
$$

Where $a_{x}$ and $a_{y}$ are the average values of the $x$ and $y$ sensors during one step. Solving for $\alpha$ yields

$$
\begin{equation*}
\alpha=\arcsin \left(\frac{\sqrt{a_{x}^{2}+a_{y}^{2}}}{g}\right) \tag{5.3}
\end{equation*}
$$

Substituting $\alpha$ in Equation (5.1) using 5.3 yields an estimation for $g_{\text {surface }}$ which is then used to run the calibration process again with the modified value.

The calibration results are validated using a validation sequence. The test sequence is composed of steps, on each step the sensor is placed in a static random orientation in space. If the calibration is accurate, the total measured acceleration on


Figure 5.7: Calibration sequence. The Shimmer is placed on a different face every few seconds. (a) 3-axis accelerometer sensor recording of the calibration process. The earth-facing(sky-facing) sensor is automatically detected for each position, and the average value is computed, corresponding to $g(-g)$.(b) 3-axis gyroscope sensor recording of the calibration process. A moving average method was used to detect the turns along each axis. Integrating the signal input over the complete turn corresponds to a 90 degrees turn value.
each step is $\hat{g}$. The Mean Squared Error $g-\hat{g}$ is the calibration sequence accuracy score.

While the sensor is static, the gyroscope signal should be zero. Hence, the offset for each gyroscope axis can simply be found by averaging over a static period. For the scaling factor, signal processing techniques were applied to accurately identify the


Figure 5.8: Validation sequence. The sensor placed in a different static 3-D position every few seconds. The magnitude of the acceleration vector is expected to be the gravity force $g$. (a) The 3 -axis accelerometer output and the calculated norm. (b) Comparison of the magnitude to the expected gravity. In the example there are fluctuations of $0.5 \mathrm{~m} / \mathrm{s}^{2}$ which might cause considerable errors.
time periods where the change in position took place. Each position change is a 90 degrees turn for one of the gyroscope sensors. Using Equation (4.4), the following
can be derived

$$
\begin{equation*}
\int_{0}^{t} y_{g, t} d t=\int_{0}^{t} a \omega_{t} d t=a \int_{0}^{t} \omega_{t} d t=a \theta_{\frac{\pi}{2}} \tag{5.4}
\end{equation*}
$$

Where $y_{g, t}$ is the gyroscope signal, $a$ is the scaling factor, $\omega_{t}$ is the angular velocity and $\theta_{\frac{\pi}{2}}$ represents a 90 degrees turn. Finally, the scaling factor $a$ can be estimated by integrating the gyroscope signal $\int_{0}^{t} y_{g, t} d t$ and extracting $a$ from the previous formula.

RSSI calibration RSSI calibration process estimates the system parameters for translating the power levels' measurements between each pair of sensor nodes to corresponding distance. RSSI measurements data is manually collected from a pre determined set of points with known distances. The calibration scheme is based on log fitting of the RSSI measurements and approximating the power offset and the channel exponent using the a-priori knowledge of the environment physical dimensions and the range of the channel exponent values. The calibration process is described in detail in [16].

### 5.5 Data Synchronization

The measurements acquired from different sensor systems cannot be used together without proper interpolation and synchronization.First, each data source has a different sampling frequency - the SHIMMER sensor operates in 208.4 Hrz, the mobile BSN node transmits data packets in 52.1 Hrz and the polaris measurements are sampled in 100 Hrz . Second, each data stream has a different time stamp, mandating the use of an accurate synchronization algorithm to create a uniform time stamp.

The synchronization between the SHIMMER mote and the mobile BSN node was accomplished by using the BSN's on-board accelerometer sensor and comparing the signal to the SHIMMER accelerometer sensor. The time delay estimation was found by calculating the cross correlation function between the signals with varying offset of one of them

$$
\begin{equation*}
c(k)=\operatorname{corr}\left(a_{b s n}(t), a_{\text {shimmer }}(t+k)\right) . \tag{5.5}
\end{equation*}
$$

Where $\operatorname{corr}(a, b)$ is the cross correlation between $a$ and $b, a_{b s n}(t)$ is the BSN accelerometer signal and $a_{\text {shimmer }}(t+k)$ is the SHIMMER accelerometer signal with a $k$ offset. The peak of the cross-correlation function is the time offset estimation.


Figure 5.9: Shimmer and BSN mobile node synchronization. Shimmer accelerometer and BSN accelerometer signals are shown after synchronization. Even though the BSN signal is not properly calibrated, the sync operation performs well.

The polaris system measurements were synchronized with the SHIMMER gyroscope measurements. The polaris system tracks both location and orientation of the moving body. Recall Equation (4.5)

$$
\begin{equation*}
\dot{\theta}_{i n, t}=\omega_{i n, t} . \tag{5.6}
\end{equation*}
$$

The angular velocity is calculated from the orientation measurements by taking the first derivative. Using the polaris estimation for angular velocity and the gyroscope measurements by the SHIMMER the time offset can be found using the cross correlation function as described in Equation (5.5).


Figure 5.10: Shimmer and Polaris system synchronization. Shimmer gyroscope and Polaris computed gyroscope signals are shown after synchronization. The polaris gyroscope signal is very noisy due to the derivation process, yet the sync operation works well. The shimmer gyroscope suffers from changing bias, most evident in the first 10 seconds.

Finally, the polaris measurements and RSSI measurements are synchronized by estimating the time offset between the location estimations from the RSSI and Polaris systems. This synchronization is redundant and its aim is to verify correctness of prior synchronizations.

### 5.6 Analysis

The filter performance analysis was split into several parts. First, Section 6.2 deals with the main goal of the filter, location estimation. A number of real-world experiments were conducted and trajectory estimations of the proposed filter were found. Comparing the results to the RSSI-based system estimations, reveals great improvement in discovering movement pattern but only slight improvement in the RMS sense (see Fig. 6.3). Second, Sections 6.3 and 6.4 analyze the filter's ability to track additional quantities such as Gyroscope and Accelerometer bias which are included in the kalman state vector. These values are not the filter's primary goal, but are interesting and valuable by themselves. Gyroscope and Accelerometer bias are artificially added to the experimental data, and the filter successfully tracks their value and detects dynamic changes. Section 6.3 further investigates the relationship between the kalman parameters, dynamic response to changes and overall filter performance. The results affirm the fact that filter parameters must be carefully set, since setting a value too low causes slow dynamic response to change, and setting a value too high degrades filter performance.

Finally, the third part (Section 6.5) validates the filter robustness to initial parameters errors. This is a critical parameter for assessing the quality of a filter, since initial errors are common in real-world scenarios. Both location,velocity and orientation errors are tested and successfully corrected.

## Chapter 6

## Results

The following sub-sections describe preliminary experimental results demonstrating the accuracy of position estimation using the inertial/RSSI tracking system.

### 6.1 Parameter identification

An example of a gyroscope and accelerometer recording is given in Figure 6.1. It shows the output signal of the shimmer sensor attached to a hand while moving on a surface. It can be seen that the gyroscope signal suffers greatly from changing bias, notice the decreasing signal magnitude in the first 5 seconds before the hand starts moving. Static measurements with the sensors laying still on the laboratory table, to obtain gyroscope and accelerometer noise, resulted in an RMS of $0.005 \frac{\mathrm{rad}}{\mathrm{s}}$ and $0.05 \frac{m}{s^{2}}$ respectively. The time constant $\delta t$ for the RSSI model as defined in Eq. (3.17) was set to 5 s . It was determined by inspecting the change in RSSI bias during an experiment as shown in Figure 6.2.

The filter initialization parameters are reported in Table 6.1. The values were determined as explained above and extensively tested in simulation and experiments.

### 6.2 Typical results

Figure 6.3 shows results typical of the hand movement experiments for the kalmanbased filter and the RSSI-only system. The experiments varied in speed, length and trajectory shape as can be seen. In each plot, both estimations are compared, with the polaris output used as reference. The inertial system estimation is not shown


Figure 6.1: Measured sensor signals during one hand movement experiment (2D movement). (a) Accelerometer output vector, only $x, y$ axis are shown since movement is 2D. (b) Gyroscope output vector. The gyroscope changing bias can be seen in the first 10 seconds before the hand starts moving.
as it significantly diverges after just a few seconds. Figure 6.4 presents the filter performance in a separate view for the x and y axis respectively. The three traces in the upper plot represent the different estimations and the traces in the lower plot represent the calculated error for each system. These plots reveal that, although both systems are similar in MMSE sense, the proposed filter achieves far great accuracy in estimating shape and pattern.

We use root mean squared error (RMSE) to compare a given trajectory $\left\{x_{n}, y_{n}, z_{n}\right\}$ and it's estimated trajectory $\left\{\hat{x_{n}}, \hat{y_{n}}, \hat{z_{n}}\right\}$

$$
\begin{equation*}
R M S E=\sqrt{\frac{1}{N} \Sigma_{n=1}^{N}\left[\left(x_{n}-\hat{x_{n}}\right)^{2}+\left(y_{n}-\hat{y_{n}}\right)^{2}+\left(z_{n}-\hat{z_{n}}\right)^{2}\right]} \tag{6.1}
\end{equation*}
$$



Figure 6.2: RSSI location error. The graph shows a changing bias with time constant of about 5 seconds. This constant is used to model the RSSI error.

| Paramter | value |
| :--- | :--- |
| $\mathbf{Q}_{v G}-$ Gyroscope sensor noise [defined in 3.5] | $0.005 \frac{\mathrm{Rad}}{\mathrm{sec}}$ |
| $\mathbf{Q}_{w g}-$ Gyroscope bias noise [defined in 3.7] | $0.05 \mathrm{Rad} / \mathrm{sec}$ |
| $\mathbf{Q}_{v A}$ - Accelerometer sensor noise [defined in 3.10] | $0.05 \frac{M}{s e c^{2}}$ |
| $\mathbf{Q}_{w a}-$ Accelerometer bias noise [defined in 3.12] | $0.05 \frac{M}{s e c^{2}}$ |
| $\mathbf{Q}_{w r}-$ RSSI white noise [defined in 3.18] | $0.1 \frac{M}{s e c}$ |
| $\delta t$ - RSSI Time Constant [defined in 3.17] | 5 sec |

Table 6.1: Kalman filter initialization parameters.

Each RMS value was calculated for the whole duration of the experiment, neglecting the first few seconds. This was to prevent errors due to initial convergence of the filter. One way to measure the convergence of the Kalman filter is through examination of the trace of the error covariance matrix. Fig 6.5 shows the trace for one of the experiments. It is noted that the sum of squared errors reaches a steady state after approximately 10 s . The table 6.2 summarizes the RMS values of the location estimation error for the INS, RSSI and the kalman filter. Each value, represents the mean of all experiments. As the table shows, the kalman filter outperforms the other system estimations. As expected, the inertial system suffers severely from drift and its location error grows indefinitely. On the other hand, the kalman filter improves the RSSI estimation only by a factor of $10-20$ percent, this is due to the inaccuracy of the inertial sensors output.


Figure 6.3: Typical results for location estimations. Each plot shows the kalman-based,RSSI-based and reference estimations. Inertial-based estimation diverges quickly and is not presented.

### 6.3 Gyroscope bias estimation

The time required for the filter to estimate the offset was tested by off-line processing of the sensor signals using an initial offset error, artificially added to the gyroscope signals prior to application of the Kalman filter. The offset error at the end of the measurement was then used as a measure for the ability of the filter to estimate the offset.

An example of gyroscope bias estimation during an experiment is given in Fig. 6.6 The system successfully estimates the bias error and handles changes in bias effectively. Fig 6.7 shows the performance of the kalman filter with a simpler model for the gyroscope, without the bias. It is clearly shown that the full kalman filter significantly outperforms the simple model. For the simple model, since the Gyroscope bias is not compensated, the orientation error grows constantly, severely degrading the filter performance.

The gyroscope bias instability value $\mathbf{Q}_{w g}$ [defined in 3.7] has critical importance

| Estimation method | Location error RMS (M) |
| :--- | :--- |
| Pure inertial system | 34.8 [for 50 seconds experiment]. Unbounded. |
| Pure RSSI-based system | 0.0927 |
| Proposed filter (inertial and RSSI) | 0.0808 |

Table 6.2: RMSE values for each estimation method. The kalman filter achieves greater accuracy, improving the RSSI estimates by 10-20\%


Figure 6.4: Typical results for location and location estimation error (LEE) decomposed for X and Y axis. Plot (a) compares the location estimation of the proposed filter with the RSSI-based estimation. The kalman filter tracks the movement and shape much better but overall RMS sense is quite similar as shown in plot (b).
for successfully handling changing bias values. Fig 6.9 shows filter behavior with several different $\mathbf{Q}_{w g}$ values. The value determines how quickly the filter can adapt to changes in the bias, setting the value too low causes poor performance during a rapid change. Unfortunately, setting the value too high comes with a price too, table 6.3 shows the RMS values for filter with different bias values. High bias value degrades overall filter performance, even if no bias is present. Hence, for optimal performance the $\mathbf{Q}_{w g}$ value must be carefully set. Similar treatment to the other parameters of the filter can be done but is not presented here due to shortage of space. In general, for optimal filter performance, intricate tuning is needed.

The gyroscope bias estimation is limited. During static periods, the bias cannot


Figure 6.5: Trace of the kalman gain matrix. The graph shows that the gain converges after 10 seconds.
be determined. Figure 6.8 shows the Gyrosocpe bias estimation error remains uninterrupted until the 8th seconds, when the movement starts. Only then, the gyroscope bias is discovered and estimated correctly and the orientation is corrected. The reason for this is inherent in the inner workings of the filter. The orientation error causes the estimated acceleration to be projected to the real world axis with a small error, causing a wrong velocity calculation. Hence, in a static zero-acceleration state, the orientation error doesn't cause any velocity errors and so cannot be discovered.

| $\mathbf{Q}_{w g}$ value | Location error RMS (M) |
| :--- | :--- |
| 0.02 (very low value) | 0.088 |
| 0.08 (average value) | 0.0808 |
| 0.3 (high value) | 0.092 |
| 1 (very high value) | 0.11 |

Table 6.3: RMSE values for each $\mathbf{Q}_{w g}$ value for a typical experiment with little bias. Filter performance is affected if an inappropriate value is used.


Figure 6.6: Filter tracking of gyroscope bias drift. Plot (a) compares the estimated Gyroscope bias with the actual changing bias. Plot (b) shows the Bias Estimation Error as a function of time.

### 6.4 Accelerometer bias estimation

The kalman filter models and successfully tracks the accelerometer bias. An example for accelerometer bias estimation is found in Fig. 6.10. The figure clearly shows the filter ability to track the changing accelerometer bias for both X and Y axis.

### 6.5 Sensitivity to initial condition errors

The system handles well initial errors for the location, velocity and orientation as seen in Figures 6.11,6.12 and 6.13 respectively. For each plot, the filter was initiated with bad initial conditions and was tested for convergence to the correct value. The values used were several scale larger than any real-world initial error might be, thus the filter is expected to recover from inaccurate initial parameters. Note, convergence


Figure 6.7: Comparison of filter performance compared to simpler model without gyroscope bias model. The top plot shows the Location Estimation Error of X $\operatorname{axis}\left(L E E_{x}\right)$ for the two filters. The LEE. The $L E E_{x}$ for the simple model grows due to growing error in the orientation estimation (OOE) as shown in the lower plot. The plots clearly demonstrate that the simpler model performance is inferior in face of constant Gyroscope bias
times may vary for the different variables, Fig. 6.13 shows the orientation estimation was the slowest to converge.


Figure 6.8: Gyroscope bias tracking during an experiment starting from a static position. The bias cannot be determined during the initial static period, it is corrected only on the 8 th second when movement starts. Note the velocity trace is plotted only for reference of movement time, $y$-axis units are not relevant.


Figure 6.9: Comparison of filter tracking changing bias for different values of $\mathbf{Q}_{w g}$ [defined in 3.7]. Setting a small value, causes slow divergence


Figure 6.10: Accelerometer bias tracking. Estimated accelerometer bias values are plotted compared to the real values. The filter converges to the correct bias values for both X and Y axis.


Figure 6.11: Location estimation for the X axis with wrong initial conditions. The plots show wrong initial X values of $0.5 \mathrm{M}, 1 \mathrm{M}$ and 1.5 M . The different traces show the filter converges after at most 20 seconds even for large errors.


Figure 6.12: Velocity estimation for the X axis with wrong initial conditions. The plots show wrong initial $X$ values of $1 \mathrm{M} / \mathrm{S}, 2 \mathrm{M} / \mathrm{S},-1 \mathrm{M} / \mathrm{S}$ and $-2 \mathrm{M} / \mathrm{S}$. The different traces show the filter converges after at most 25 seconds even for large errors.


Figure 6.13: Orientation estimation error. Orientation is represented as a counter clock-wise rotation in the XY plane. The plots show wrong initial values of 1 RAD, 2 RAD, -1 RAD and -2 RAD. For large errors, the convergence occurs only after considerable time.

## Chapter 7

## Discussion and Future Work

The filter parameter initialization reported in Table 6.1 is found to work well after running an extensive number of tests in the presence of the simulated disturbances. Of course, for different trajectories and time-varying disturbances, different sets of filter parameters would be better. The discussion in Section 6.3 outlines the major implications on overall filter performance caused by setting an inappropriate gyroscope bias instability value. The same treatment can be done to the other 15 variables in the kalman state vector, hence finding the optimal filter parameters is an intricate and constant task. Not only sensor noise parameters are taken into considerations, but also the shape of the path and velocity of the movement (which affect RSSI bias modeling), even SHIMMER's battery depletion might increase the bias instability factor.

The proposed filter outperforms the RSSI-based system but not significantly. This is due to a number of reasons :

- The Root Mean Square criterion does not tell the whole story. As shown in Figure 6.4, the filter tracks the movement pattern and shape much better then the RSSI-based algorithm, a different criterion might reward this behavior more significantly.
- The use of noisy and bias unstable sensors in the real-world experiments. As Figure 6.1 shows, that SHIMMER sensors suffer from severe bias drift which degrades filter performance. In future work, these experiments will be repeated with the newer generation sensors, e.g. SHIMMER2, possibly integrating the inertial sensors with more drift-free sensors such as magnetometers. Simulation results using improved inertial sensors achieve far better results, as seen in this
table 7.1 compared to the initial table 6.2.
- RSSI-based estimations errors are slow changing and are not optimally modeled with a gaussian white noise process as the kalman filter requires. A possible solution, is to further investigate in the future other types of filters, such as the particle filter which are more capable with handling such errors.
- Sampling rate considerations are not taken into account. Any real-world system will restrict the RSSI sampling rate due to energy consumption considerations. Lower sampling rates will greatly degrade the RSSI-based estimation accuracy, becoming more dependent of the successful fusion with inertial sensors. Future work will investigate the filter performance under different sampling rates constraints.

There are a number of ways the current sensor setup can be improved. First, incorporating newer inertial sensors such as the SHIMMER2 motes and integrating them with aiding sensors such as magnometers will greatly improve results [28]. Second, the presented kalman filter successfully combines inertial data with RSSI-based estimations, but handles each system in a black box manner. Borrowing terms from the GPS/INS world, the presented filter is a closed-loop design with no feedback [81]. The future work will be addressed to apply the ideas presented here with an open-loop design, using the RSSI bias estimation as a feedback to the RSSI-based estimator. As in the realm of GPS/INS, where open-loop designs allow for intimate cooperation between the systems, it applies here too. The RSSI-based system can use the information to detect in real-time shadowing effects and learn how to better compensate for them.

Although the prototype system works under lab conditions, the path to a useful tracking system for Body Sensor Networks is long and hard. Lets review some of the major milestones left for future research:

From 2D to 3D The RSSI-based system is currently only implemented in 2D. Developing the system to work with more anchor nodes and supply 3D estimations is critical.

BSN implementation The system should be able to run in real-time on-board the moving object. This requires implementing the whole system on a single wireless mote, including hardware (current setup uses two different platforms for INS and RSSI) and software (code in TinyOs and not matlab).

Dynamic anchor nodes RSSI-based algorithms assume the anchor nodes are static, whereas in the realm of human motion tracking, the body moves and so the anchor nodes move too. This has two major implications, first the RSSI-based algorithm must be verified to work in a dynamic environment and second the inertial system must implement algorithms for deferential inertial tracking between the limbs and the body. The required system is similar to the work of Foxlin et al. [36] which implemented differential inertial tracking between helmet-mounted and aircraft-mounted inertial sensors as part of the development of FlightTracker, a system for cockpit enhanced vision.

| Estimation method | Location error RMS (M) |
| :--- | :--- |
| Pure inertial system | 3.76 [for 50 seconds experiment]. Unbounded. |
| Pure RSSI-based system | 0.058 |
| Proposed filter (inertial and RSSI) | 0.042 |

Table 7.1: RMSE values for each estimation method from simulation results

## Chapter 8

## Conclusions

This thesis presents the design, implementation, simulations and real-world experimental results of a sensor-fusion Kalman filter for real-time human body motion tracking using inertial/RSSI sensors. The complete design of the filter was presented, detailing the inner workings of each sub system and the full derivation of the complementary kalman filter used for sensor fusion. Also, a new and simple derivation for the orientation estimation kalman filter based on Luinge et at. work [43] was presented (see Section 4.1.2). A real-world experimental setup was built, including a state-of-the-art optical tracking device used as reference. Finally, realworld experiments of hand movement were performed but only in a controlled, 2D environment and with a custom made RSSI-based setup. Implementation of the filter on existing BSNs with full human motion testing in real-life environment is left for future research.

The Kalman filter design presented in this work is the result of two years of effort into close-proximity RSSI-based tracking systems. Continuing the work of Blumrozen et al. in [56] and [16], this research expands their work to the realm of Inertial Body Sensor Networks. Although this system is merely an oversimplified prototype (as discussed in Section 7), this filter holds great potential to be implemented by numerous existing and future Body Sensor Networks (BSNs) with inertial sensors and wireless capability, as aiding the tracking algorithm with RSSI data comes with no added hardware costs.

## Bibliography

[1] Eric Foxlin. Chapter 7 . motion tracking requirements and technologies. Environment, pages 163-210, 2002.
[2] Benny P. L. Lo and Guang-Zhong Yang. Key Technical Challenges and Current Implementations of Body Sensor Networks. In International Workshop on Wearable and Implantable Body Sensor Networks, April 2005.
[3] B.R.; McGrath M.J.; O'Shea T.J.; Kuris B.; Ayer S.M.; Stroiescu F.; Cionca V. Burns, A.; Greene. Shimmer a wireless sensor platform for noninvasive biomedical research. In Sensors Journal, IEEE, September 2010.
[4] K Tong and M H Granat. A practical gait analysis system using gyroscopes. Medical Engineering and Physics, 21(2):87-94, 1999.
[5] Kamiar Aminian, B Najafi, C Bla, P-F Leyvraz, and Ph Robert. Spatio-temporal parameters of gait measured by an ambulatory system using miniature gyroscopes. Journal of Biomechanics, 35(5):689-699, 2002.
[6] R Williamson and B J Andrews. Sensor systems for lower limb functional electrical stimulation (fes) control. Medical Engineering and Physics, 22(5):313-25, 2000.
[7] K Y Tong and M H Granat. Virtual artificial sensor technique for functional electrical stimulation. Medical Engineering and Physics, 20(6):458-468, 1998.
[8] P H Veltink, P Slycke, J Hemssems, R Buschman, G Bultstra, and H Hermens. Three dimensional inertial sensing of foot movements for automatic tuning of a two-channel implantable drop-foot stimulator. Medical Engineering and Physics, 25(1):21-28, 2003.
[9] A T Willemsen, F Bloemhof, and H B Boom. Automatic stance-swing phase detection from accelerometer data for peroneal nerve stimulation. IEEE Transactions on Biomedical Engineering, 37(12):1201-1208, 1990.
[10] M J Mathie, A C F Coster, N H Lovell, and B G Celler. Detection of daily physical activities using a triaxial accelerometer. Medical and Biological Engineering and Computing, 41(3):296-301, 2003.
[11] F Foerster. Detection of posture and motion by accelerometry: a validation study in ambulatory monitoring. Computers in Human Behavior, 15(5):571-583, 1999.
[12] Carlijn V C Bouten, Karel T M Koekkoek, Maarten Verduin, Rens Kodde, and Jan D Janssen. A triaxial accelerometer and portable data processing unit for the assessment of daily physical activity. IEEE Transactions on Biomedical Engineering, 44(3):136-147, 1997.
[13] Am Sabatini. Inertial sensing in biomechanics: a survey of computational techniques bridging motion analysis and personal navigation, volume 1, page 70100. Idea Group Pubilishing: Hershey, PA, USA, 2006.
[14] N Yazdi, F Ayazi, and K Najafi. Micromachined inertial sensors. Proceedings of the IEEE, 86(8):1640-1659, 1998.
[15] Xiaoping Yun, Eric R Bachmann, Hyatt Moore, and James Calusdian. Selfcontained position tracking of human movement using small inertial/magnetic sensor modules. Proceedings 2007 IEEE International Conference on Robotics and Automation, (April):2526-2533, 2007.
[16] Tal Anker Danny Dolev Boris Rubinsky Gaddi Blumrozen, Bracha Hod. Enhancing rssi-based tracking accuracy in wireless sensor networks. To be published on ACM Transactions on Sensor Networks, 2011.
[17] N Yazdi, F Ayazi, and K Najafi. Micromachined inertial sensors. Proceedings of the IEEE, 86(8):1640-1659, 1998.
[18] XSens. Xsens mtx inertial sensor, 2011.
[19] Jawbone Inc. Wireless wristband for a healthier life. http://http://jawbone. com/up, 2011. [Online; accessed 1-December-2011].
[20] H Zhou and H Hu. Human motion tracking for rehabilitationa survey. Biomedical Signal Processing And Control, 3(1):1-18, 2008.
[21] Inertial Navigation. Inertial navigation forty years of evolution. Evolution, 13(3):140-149, 1998.
[22] M F Golnaraghi. A vector-based gyro-free inertial navigation system by integrating existing accelerometer network in a passenger vehicle. PLANS 2004 Position Location and Navigation Symposium IEEE Cat No04CH37556, pages 234-242, 2004.
[23] B Barshan and H F Durrant-Whyte. Inertial navigation systems for mobile robots. IEEE Transactions on Robotics and Automation, 11(3):328-342, 1995.
[24] Suya You, Ulrich Neumann, and Ronald Azuma. Hybrid inertial and vision tracking for augmented reality registration, volume 19, pages 260-267. IEEE, 1999.
[25] Greg Welch. Motion tracking : No silver bullet, but a respectable. IEEE Computer Graphics and Applications, (December), 2002.
[26] Oliver J. Woodman, C Oliver J. Woodman, and Oliver J. Woodman. An introduction to inertial navigation, 2007.
[27] Benoit Huyghe, Jan Doutreloigne, and Jan Vanfleteren. 3d orientation tracking based on unscented kalman filtering of accelerometer and magnetometer data. 2009 IEEE Sensors Applications Symposium, (1):148-152, 2009.
[28] Eric Robert Bachmann. Inertial and magnetic tracking of limb segment orientation for inserting humans into synthetic environments. PhD thesis, Citeseer, 2000.
[29] D Roetenberg, H Luinge, and P Veltink. Inertial and magnetic sensing of human movement near ferromagnetic materials. The Second IEEE and ACM International Symposium on Mixed and Augmented Reality 2003 Proceedings, pages 268-269, 2006.
[30] Y Tao, H Hu , and H Zhou. Integration of vision and inertial sensors for 3d arm motion tracking in home-based rehabilitation. The International Journal of Robotics Research, 26(6):607-624, 2007.
[31] J A Corrales, F A Candelas, and F Torres. Hybrid tracking of human operators using imu/uwb data fusion by a kalman filter. Proceedings of the 3rd international conference on Human robot interaction HRI 08, page 193, 2008.
[32] L. Sher. Personal inertial navigation system (pins). 1996.
[33] Daniel Vlasic, Rolf Adelsberger, Giovanni Vannucci, John Barnwell, Markus Gross, Wojciech Matusik, and Jovan Popovic. Practical motion capture in everyday surroundings. ACM Transactions on Graphics, 26(3):35, 2007.
[34] P Corke, J Lobo, and J Dias. An introduction to inertial and visual sensing. The International Journal of Robotics Research, 26(6):519-535, 2007.
[35] Eric Foxlin and Leonid Naimark. VIS-Tracker: A Wearable Vision-Inertial SelfTracker. 2003.
[36] Eric Foxlin, Yury Altshuler, Leonid Naimark, and Mike Harrington. Flighttracker : A novel optical / inertial tracker for cockpit enhanced vision. Third IEEE and ACM International Symposium on Mixed and Augmented Reality, (Ismar):212221, 2004.
[37] J Lobo and J Dias. Vision and inertial sensor cooperation using gravity as a vertical reference. IEEE Transactions on Pattern Analysis and Machine Intelligence, 25(12):1597-1608, 2003.
[38] Toshiharu Mukai and Noboru Ohnishi. The recovery of object shape and camera motion using a sensing system with a video camera and a gyro sensor. Proceedings of the Seventh IEEE International Conference on Computer Vision, 1(1):411-417 vol.1, 1999.
[39] Francesco Panerai, Giorgio Metta, and Giulio Sandini. Visuo-inertial stabilization in space-variant binocular systems. Robotics and Autonomous Systems, 30(1-2):195-214, 2000.
[40] E Foxlin. Inertial head-tracker sensor fusion by a complementary separate-bias kalman filter. Proceedings of the IEEE 1996 Virtual Reality Annual International Symposium, 96:185-194,, 1996.
[41] Eric Foxlin. Pedestrian tracking with shoe-mounted inertial sensors. IEEE Computer Graphics and Applications, 25(6):38-46, 2005.
[42] Xiaoping Yun, Eric R Bachmann, Hyatt Moore, and James Calusdian. Selfcontained position tracking of human movement using small inertial/magnetic sensor modules. Proceedings 2007 IEEE International Conference on Robotics and Automation, (April):2526-2533, 2007.
[43] H J Luinge and P H Veltink. Measuring orientation of human body segments using miniature gyroscopes and accelerometers. Medical and Biological Engineering and Computing, 43(2):273-282, 2005.
[44] Xiaoping Yun and Eric R Bachmann. Design, implementation, and experimental results of a quaternion-based kalman filter for human body motion tracking. IEEE Transactions on Robotics, 22(6):1216-1227, 2006.
[45] A D Young. Use of body model constraints to improve accuracy of inertial motion capture. 2010 International Conference on Body Sensor Networks, pages 180-186, 2010.
[46] Huiyu Zhou, Huosheng Hu, and A Motivation. Kinematic model aided inertial motion tracking of human upper limb. 2005 IEEE International Conference on Information Acquisition, pages 150-155, 2005.
[47] Ferial Shayeganfar, Amin Anjomshoaa, and A Min Tjoa. A smart indoor navigation solution based on building information model and google android. Methodology, 5105:1050-1056, 2008.
[48] Hui Liu, Houshang Darabi, Pat Banerjee, and Jing Liu. Survey of wireless indoor positioning techniques and systems. IEEE Transactions on Systems Man and Cybernetics Part C Applications and Reviews, 37(6):1067-1080, 2007.
[49] Localization of wireless sensor networks with a mobile beacon. 2004 IEEE International Conference on Mobile Adhoc and Sensor Systems IEEE Cat No04EX975, (July).
[50] Tsenka Stoyanova, Fotis Kerasiotis, Aggeliki Prayati, and George Papadopoulos. Evaluation of impact factors on rss accuracy for localization and tracking applications. Proceedings of the 5th ACM international workshop on Mobility management and wireless access MobiWac 07, $\operatorname{Im}(17): 9,2007$.
[51] F V F De Lima and C M Furukawa. Development and testing of an acoustic positioning system - description and signal processing, volume 1, pages 849-852 vol.1. 2002.
[52] Andreas Savvides, Heemin Park, and Mani B Srivastava. The bits and flops of the n-hop multilateration primitive for node localization problems. Proceedings of the 1st ACM international workshop on Wireless sensor networks and applications WSNA 02, pages 112-121, 2002.
[53] Andreas Savvides, Heemin Park, and Mani B Srivastava. The n-hop multilateration primitive for node localization problems. Mobile Networks and Applications, pages 443-451, 2003.
[54] P Bahl and V N Padmanabhan. Radar: an in-building rf-based user location and tracking system. Proceedings IEEE INFOCOM 2000 Conference on Computer Communications Nineteenth Annual Joint Conference of the IEEE Computer and Communications Societies Cat No00CH37064, 2(c):775-784, 2000.
[55] Mohit Saxena, Puneet Gupta, and Bijendra Nath Jain. Experimental analysis of rssi-based location estimation in wireless sensor networks. 2008 3rd International Conference on Communication Systems Software and Middleware and Workshops COMSWARE 08, pages 503-510, 2008.
[56] Gaddi Blumrosen, Bracha Hod, Tal Anker, Danny Dolev, and Boris Rubinsky. Continuous close-proximity rssi-based tracking in wireless sensor networks. In $B S N$, pages 234-239, 2010.
[57] Lasse Klingbeil, Richard Reiner, Michailas Romanovas, Martin Traechtler, and Yiannos Manoli. Multi-modal sensor data and information fusion for localization in indoor environments. Spectrum, pages 187-192, 2010.
[58] Andreas Fink, Helmut Beikirch, Matthias Vo?, and Christian Schr. RSSI-based Indoor Positioning using Diversity and Inertial Navigation, pages 15-17. Number September. 2010.
[59] Fr?d?ric Evennou and Fran?ois Marx. Advanced integration of wifi and inertial navigation systems for indoor mobile positioning. EURASIP Journal on Advances in Signal Processing, 2006:1-12.
[60] John Bortz. A new mathematical formulation for strapdown inertial navigation. Ieee Transactions On Aerospace And Electronic Systems, AES-7(1):61-66, 1971.
[61] M S Grewal, L R Weill, and A P Andrews. Global positioning systems, inertial navigation, and integration, page 364. Wiley-Interscience, 2007.
[62] Robert Grover Brown and Patrick Y C Hwang. Introduction to Random Signals and Applied Kalman Filtering, volume 2, page 401. John Wiley and Sons, 1997.
[63] G Mao, B Fidan, and B Anderson. Wireless sensor network localization techniques. Computer Networks, 51(10):2529-2553, 2007.
[64] Eli Brookner. Tracking and Kalman Filtering Made Easy, volume 1, page 107. John Wiley and Sons, Inc., 1998.
[65] Robert Grover Brown and Patrick Y C Hwang. Introduction to Random Signals and Applied Kalman Filtering, volume 2, page 195. John Wiley and Sons, 1997.
[66] Dan Simon. Optimal State Estimation, volume 2, page 26. John Wiley and Sons, Inc., 2006.
[67] Dan Simon. Optimal State Estimation, volume 2, page 189. John Wiley and Sons, Inc., 2006.
[68] Peter Henrici. Introduction to applied mathematics (gilbert strang). SIAM Review, 28(4):590, 1986.
[69] Robert Grover Brown and Patrick Y C Hwang. Introduction to Random Signals and Applied Kalman Filtering, volume 2, page 392. John Wiley and Sons, 1997.
[70] K Pahlavan and J P Makela. Indoor geolocation science and technology. IEEE Communications Magazine, 40(2):112-118, 2002.
[71] S Hara, K Yanagihara, J Taketsugu, K Fukui, S Fukunaga, and K Kitayama. Propagation characteristics of ieee 802.15 .4 radio signal and their application for location estimation. 2005 IEEE 61st Vehicular Technology Conference, 1(c):97101, 2005.
[72] W. LIAO and Y LEE. A lightweight localization scheme in wireless sensor networks. ICWMC06, 2006.
[73] Dan Simon. Optimal State Estimation, volume 2, page 19. John Wiley and Sons, Inc., 2006.
[74] S. E. Leis. Ndi-tb-0005: Polaris calibration performance and methodology, rev. 002. tech. rep. Northern Digital Inc., 1996.
[75] Intel Corporation. The shimmer sensor node platform. 2006. [Online]. Available: http://www.eecs.harvard.edu/ mdw/proj/codeblu.
[76] B. Lo, S. Thiemjarus, R. King, and G.Z. Yang. Body sensor network-a wireless sensor platform for pervasive healthcare monitoring. In The 3rd International Conference on Pervasive Computing, volume 13. Citeseer, 2005.
[77] AS Chipcon. CC2420 2.4 GHz IEEE 802.15. 4/ZigBee-ready RF Transceiver. Chipcon AS, Oslo, Norway, 4, 2004.
[78] Sohan Ranjana Brian Blakec Kevin Clearya, Luis Ibanezb. Igstk: a software toolkit for image-guided surgery applications. Computer Aided Radiology and Surgery, 2004.
[79] Oliver J. Woodman. An introduction to inertial navigation. 2007.
[80] Standard specification format guide and test procedure for single-axis interferometric fiber optic gyros. IEEE Std 962, 2003.
[81] R T Kelley, I N Katz, and C A Bedoya. Design, development and evaluation of an Ada coded INS/GPS open loop Kalman filter, volume 4, page 382388. IEEE, 1990.

