Perfect Competition and Competitive Equilibria in Economies of Indivisible Objects

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Abstract

It is shown that, in a pure exchange economy of indivisible objects, if the valuation of no trader exhibits complementarity and if there are many traders, none of them is influential enough to influence the market by himself, there exists a competitive (i.e., Walrasian) equilibrium. This result should be compared, on one hand, with that of Kelso and Crawford, who prove the existence of such an equilibrium only among traders that satisfy a much stronger substitutes condition, but without any assumption on the number of traders, and, on the other hand, with that of Aumann on the existence of a competitive equilibrium in an economy of divisible objects amongst a continuum of traders.

1 Background

Even though the notion of a competitive equilibrium dates back to Walras [9], it was Wald [8] who brought to the attention of economists the problem of proving rigorously the existence of such equilibria in a market. Intuitively, the existence of competitive equilibria, or their enforcement in market, depends on each trader renouncing the idea of influencing (downwards) the prices. This typically happens in markets with a large number of traders.

For economies of divisible objects the existence of a competitive equilibrium has been proved under two very different types of assumptions. Arrow, Debreu [1] and many others assume that the valuation of every trader is concave, a very restrictive and quite unintuitive assumption. Aumann [3] assumes nothing about the form of the valuations of the individual traders, but assumes that there is a continuum of traders, implying that none of them can individually influence the prices.

For economies of indivisible objects, Kelso and Crawford [6] proved the existence of a competitive equilibrium under a very restrictive assumption: they assume that the valuation of each trader is substitutes (gross-substitutes in their terminology). In this paper, we consider markets of indivisible objects under perfect competition, i.e., no trader alone can influence the outcome or the prices. We propose a formal definition of perfect competition that does not require considering an infinite number of traders. We show that not all perfectly competitive economies possess a competitive equilibrium, but that every such economy in which no trader exhibits complementarity does. The no-complementarity assumption is much less restrictive than Kelso and Crawford's substitutability assumption: by a result of [7], substitutes valuations have measure zero whereas no-complement valuations have a positive measure. The results of Bikhchandani and Mamer [4] are used in the proof.

2 Setting and Plan

2.1 Setting

The setting is the one considered in previous works by Bikhchandani and Mamer [4] and Gul and Stacchetti [5]: indivisible objects, private values, quasi-linear utilities, no externalities, free disposal and normalization. We also assume that all agents have enough money to perform all trades they find profitable. In this setting there is no need to consider initial endowments, one may assume that there is only one seller whose possesses all the objects initially but attaches them a zero value and a set of buyers whose initial endowment is empty, except for a large sum of money.

Formally, a finite non-empty set of *indivisible* objects Ω and a finite non-empty set I of agents are given. The preferences of agent $i \in I$ are given by a valuation: v_i that gives every bundle $D \subseteq \Omega$ a real nonnegative value $v_i(D)$. These valuations all satisfy:

- **Free disposal:** $v_i(A) \leq v_i(B)$, whenever $A \subseteq B$.
- **Normalization:** $v_i(\emptyset) = 0$.

Definition 1 *An economy E is a tuple $E = \langle \Omega; I; v_i : i \in I \rangle$ as above. Given an economy E, subsets $X \subseteq \Omega$ and $P \subseteq I$ define the sub-economy E_X^P of E where $E_X^P = \langle X; P; v'_i : i \in P \rangle$ where $v'_i(A) = v_i(A)$ for any $A \subseteq X$ and $i \in P$.*

We shall need to consider valuations that exhibit no complementarity.

Definition 2 *A valuation $v : 2^\Omega \longrightarrow \mathcal{R}_+$ is said to be* complement-free *iff for any $A, B \subseteq \Omega$ such that $A \cap B = \emptyset$, one has $v(A) + v(B) \geq v(A \cup B)$. An economy E all the valuations of which are complement-free is said to be complement-free.*

Note that sub-economies of a complement-free economy are complement-free. The relation between complement-free, submodular, substitutes valuations and other related families is considered at length in [7].

2.2 Plan of this paper

Section 3 recalls the classical definition of a competitive, i.e., Walrasian, equilibrium in a discrete pure exchange economy. Following [4], Section 4 describes the problem of finding the optimal allocation in a pure exchange discrete economy as an Integer Programming problem, and defines the prices corresponding to the economy by reference to the dual of its linear relaxation. Section 5 presents an original analysis of the idea of perfect competition. It turns out that, for indivisible goods, one may easily define perfectly competitive markets, i.e., markets in which no single trader can influence the final outcome by himself. It is shown that markets in which each type of trader is represented a large number of times are perfectly competitive. A similar analysis for markets of divisible goods is essentially impossible and, for such markets, one must resort, as did Aumann, to an infinity (even a continuous infinity) of traders to be able to consider perfectly competitive markets. Section 6 presents a technical result about economies in which no agent exhibits complementarity. Section 7 presents the main result of this work: the existence of a competitive equilibrium is guaranteed in a complement-free, fully competitive economy. Sections 8 and 9 test the limits of competitive equilibria outside our assumptions. Section 10 concludes.

3 Competitive Equilibria

The following definition is classical. It will not, in fact, be needed. It is included only for completeness' sake.

Definition 3 *A competitive equilibrium for an economy E comprises a partial allocation $a : \Omega \longrightarrow I \cup \{unalloc\}$ and a price vector $\langle p_j \rangle$, $j \in \Omega$ such that:*

$$v_i(a^{-1}(i)) - \sum_{j \in a^{-1}(i)} p_j \geq v_i(D) - \sum_{j \in D} p_j \text{ for any } i \in I, D \subseteq \Omega. \tag{1}$$

and

$$p_j = 0, \ \forall j \text{ such that } a(j) = unalloc. \tag{2}$$

4 Optimal allocations: integral and fractional

Following Bikhchandani and Mamer [4], we shall define a number of optimization problems. The notions that will be defined now will help us to define precisely the notion of perfect competition.

We assume that an economy $E = \langle \Omega; I; v_i : i \in I \rangle$ is given. Let $m = \mid \Omega \mid$ be the number of items and $n = \mid I \mid$ the number of traders. The integer programming problem **IP**(E) below describes the efficient allocation of the items of Ω among the traders. It uses $n \times 2^m$ variables and $n + m$ constraints.

Integer Program (IP(E)):

Maximize

$$\sum_{i \in I} \sum_{D \subseteq \Omega} x^i_D \, v_i(D) \tag{3}$$

under the constraints

$$\sum_{D \mid j \in D} \sum_{i \in I} x^i_D \ \leq \ 1, \text{ for all } j \in \Omega, \tag{4}$$

$$\sum_{D \subseteq \Omega} x^i_D \ \leq \ 1 \text{ for all } i \in I, \text{ and} \tag{5}$$

$$x^i_D \in \{0, 1\}, \text{ for all } i \in I, D \subseteq \Omega. \tag{6}$$

The optimal solution set to **IP(E)** is the set of efficient allocations in $E = \langle \Omega; I; v_i : i \in I \rangle$. Constraint 5 expresses the fact that each agent gets at most one bundle. The reader may feel that one should have required that every agent gets exactly one bundle. But one easily sees that both formulations are equivalent in the sense that to any feasible solution of our problem corresponds a feasible solution in which every agent gets a bundle and that has the same value. Indeed an agent that gets no bundle could equivalently be given the empty bundle without violating constraint 4 or changing the objective function since $v_i(\emptyset) = 0$. The value of an optimal feasible solution to **IP(E)** will be denoted $I(E)$.

The linear program **LPR**(E) is the relaxation of **IP**(E).

Linear Programming Relaxation (LPR(E)):

Maximize

$$\sum_{i \in I} \sum_{D \subseteq \Omega} x^i_D \, v_i(D) \tag{7}$$

under the constraints

$$\sum_{D | j \in D} \sum_{i \in I} x^i_D \ \leq \ 1, \ \text{for all } j \in \Omega, \tag{8}$$

$$\sum_{D \subseteq \Omega} x^i_D \ \leq \ 1 \text{ for all } i \in I, \ \text{and} \tag{9}$$

$$x^i_D \in [0, 1], \ \text{for all } i \in I, D \subseteq \Omega. \tag{10}$$

The optimal solution set to **LPR**(E) can be viewed as the set of efficient *fractional* allocations in the economy E'. In the economy E' agents can be allocated fractional bundles of the form: $\alpha \in [0, 1]^{2^\Omega}$ as long as $\sum_{D \subseteq \Omega} \alpha(D) \leq 1$ and as, for each item, the sum of the fractions of it that are allocated does not exceed one. Agent i values bundle α at: $\sum_{D \subseteq \Omega} \alpha(D) v_i(D)$. Constraint 9 expresses the fact that the fractions of bundles obtained by one agent sum up to at most 1. As noticed above about **IP**(E), we could equivalently have required that each agent's fractions add up to exactly 1, since the slack can be filled in by fractions of the empty bundle without violating other constraints or changing the objective function. In other terms, we may consider only optimal solutions of **LPR**(E) in which all the constraints of 9 are saturated. The value of an optimal solution to **LPR**(E) will be denoted $L(E)$. Clearly $I(E) \leq L(E)$.

5 Perfect Competition

We shall now define perfect competition. Intuitively, an economy is perfectly competitive if no trader can, alone, bring the the price of an object down by refusing to buy it at the market price. In a perhaps surprising way, even though perfect competition has been considered early on in economic thought as a necessary condition for justifying the assumption that agents are price-takers, the notion itself has never been formalized. Our consideration of discrete economies (of indivisible goods) offers a straightforward definition of perfect competition. On the contrary, in a market of infinitely divisible goods, typically (e.g., for buyers exhibiting no complementarity), no market with a finite number of buyers can be perfectly competitive. The reason is that, in an optimal allocation, an unbounded number of buyers may get a positive quantity of the goods. In a market of m indivisible goods, no more than m buyers can get a non-empty bundle.

First, we shall define fully competitive economies, and then, we shall show that many exchanges are fully competitive: with some abuse, markets with a large number of complement-free traders are fully competitive.

5.1 Fully competitive economies

We note that, in our setting, the most efficient way for a trader to lower a price is to refuse to participate in the exchange, and therefore perfect competition implies that, for any trader $i \in I$, the prices obtained in E are the same as those obtained in $E_X^{I-\{i\}}$. But what are prices? It is a deep insight of [4] that prices are defined by considering the dual to LPR(E).

The dual of **LPR(E)** will be described now.

Dual Linear Program (DLP(E)):

Minimize

$$\sum_{j \in \Omega} p_j + \sum_{i \in I} \pi_i \tag{11}$$

under the constraints

$$p_j \geq 0, \pi_i \geq 0 \text{ for all } j \in \Omega, i \in I, \text{ and} \tag{12}$$

$$\sum_{j \in D} p_j + \pi_i \geq v_i(D), \text{ for all } D \subseteq \Omega \text{ and } i \in I. \tag{13}$$

The value of an optimal solution to **DLP(E)** will be denoted $D(E)$. By the Linear Programming Duality theorem, $D(E) = M(E)$.

The p_j, for $j \in \Omega$ are the price variables and their values in an optimal solution of DLP(E) are, in fact, a set of *right* prices, even though they, as we shall see, do not always define a competitive equilibrium. Influencing the prices is therefore equivalent to influencing the value $M(E)$.

We shall say that, in an economy E, agent $i \in I$ is *uninfluential* if the optimal *fractional* value can be obtained without it. The definition is straightforward, once noticed that we want to consider the fractional optimum, not the integral optimum.

Definition 4 *In an economy $E = \langle \Omega; I; v_i : i \in I \rangle$, an agent $j \in I$ is said to be* uninfluential *iff $L(E) = L(E_{\Omega}^{I-\{j\}})$. Otherwise j is said to be* influential.

Example 1 *Consider the economy E_1 with three objects $\{a, b, c\}$ and three agents in which all agents have the same symmetric valuation v_1 in which: $v_1(\emptyset) = 0$, $v_1(D) = 1$ for $\mid D \mid= 1$ and $v_1(D) = 10$ for $\mid D \mid\geq 2$. The optimal integral allocation has value 11 but the optimal fractional allocation allocates one half of a set of two elements to each one of the agents for a value of 15. In the economy E_1, each agent is influential. Notice, though, that an integral optimum may be obtained with one trader only.*

Definition 5 *An economy $E = \langle \Omega; I; v_i : i \in I \rangle$ is said to be* competitive *iff no agent is influential.*

Example 2 *Consider the economy E_2 with three objects and four agents in which all agents have the same symmetric valuation v_1 described in Example 1. The fractional optimum is still 15 and the economy E_2 is competitive.*

To show the existence of a competitive equilibrium, we need a stronger property than competitiveness. We need sub-economies to be competitive too, at least those sub-economies derived from E by taking away from E at least as many items of Ω than agents of I.

Definition 6 *An economy $E = \langle \Omega; I; v_i : i \in I \rangle$ is said to be* fully competitive *iff for every $A \subseteq \Omega$ and any $P \subseteq I$ such that $\mid P \mid\leq\mid A \mid$ the economy $E_{\Omega-A}^{I-P}$ is competitive.*

Example 3 *Consider the economy E_3 with four objects: $\{a, b, c, d\}$ and three agents in which all agents have the same valuation described by: for any $D \subseteq \{a, b, c\}$, $v(D) = v_1(D)$, for any $D \subset \{a, b, c\}$, $v(D \cup \{d\}) = v(D)$ and $v(\Omega) = 100$. The economy E_3 is competitive since the fractional optimum is 100 and can be obtained with only one of the agents. But E_3 is not fully competitive since $E^I_{\{a,b,c\}}$ is not competitive as shown in Example 1.*

Example 4 *The economy E_2 of Example 2 is fully competitive since the sub-economies on two items and three agents are competitive.*

5.2 Existence of fully competitive economies

One may ask whether interesting fully competitive economies exist. The following theorem shows that, with some abuse of language, complement-free economies with a large number of traders are fully competitive. Specifically, it shows that, if every type is represented at least $m + 1$ times, where m is the number of items, then the economy is fully-competitive.

Theorem 1 *Let $E = \langle \Omega; I; v_i : i \in I \rangle$ be a* complement-free *economy with $m =\mid \Omega \mid$. If for any $i_0 \in I$, there are $m + 1$ distinct agents: $i_0, \ldots i_m$ such that, for any $l = 0, \ldots, m$, $v_{i_l} = v_{i_0}$, then the economy E is fully competitive.*

Proof: Since the conditions of the theorem clearly hold for any sub-economy $E^{I'}_{\Omega'}$ with $\mid I - I' \mid \leq \mid \Omega - \Omega' \mid$, it is enough to show that E is competitive. Let x^i_D, $i \in I, D \subseteq \Omega$, be an optimal fractional allocation. We shall define an optimal fractional allocation y^i_D by reallocating the goods in-between all agents of the same type. Consider all the k agents that have the same valuation ($k \geq m + 1$): $i_1, \ldots i_k$. For every $j \in \Omega$, let a_j be the total amount of good j allocated to the agents $i_1, \ldots i_k$.

$$a_j = \sum_{l=1}^{k} \sum_{D \subseteq \Omega, j \in D} x^{i_l}_D.$$

Let us reallocate those quantities by giving all the quantity a_j to the same agent, who receives nothing else. Formally define $y^{i_j}_{\{j\}} = a_j$ and all other $y^{i_l}_D$ to be zero. Since $k \geq m$, no agent gets a positive amount of more than one good and since all valuations are complement-free, the allocation y obtained is as good as x and optimal. But, since $k \geq m + 1$, the agent i_{m+1} gets nothing and no agent is influential: i_{m+1} can replace any one of the m first agents. ∎

6 Basic result for complement-free economies

Before we go on defining perfect competition, we shall prove the main technical lemma about complement-free economies that we shall need. Given a complement-free economy E, a trader $j \in I$ and an item $a \in \Omega$, we want to compare the values I and L for the economies E and its sub-economies $E^{I}_{\Omega-\{a\}}$ and $E^{I-\{j\}}_{\Omega-\{a\}}$.

Lemma 1 *If $E = \langle \Omega; I; v_i : i \in I \rangle$ is a complement-free economy, there is a trader $j \in I$ and an item $a \in \Omega$ such that*

$$I(E) - I(E^{I-\{j\}}_{\Omega-\{a\}}) \;\geq\; L(E) - L(E^{I}_{\Omega-\{a\}}). \tag{14}$$

Intuitively: removing both a and j from the integral economy causes a loss of efficiency at least equal to the loss caused by removing a from the fractional economy.

Proof: We shall prove that, for any item $a \in \Omega$, there is a trader $j_a \in I$ with the property claimed. Let $a \in \Omega$ be an arbitrary item. Take, as j_a any trader whose valuation for the singleton $\{a\}$ is maximal among all traders:

$$v_{j_a}(\{a\}) \;\geq\; v_i(\{a\}),\ \forall i \in I. \tag{15}$$

Consider the optimal solution to $\mathbf{IP}(E^{I-\{j_a\}}_{\Omega-\{a\}})$. One may easily extend it to an integral solution of $\mathbf{IP}(E)$ by allocating a to j_a. In this new allocation j_a gets the singleton a and therefore the value of this solution is: $I(E^{I-\{j_a\}}_{\Omega-\{a\}}) + v_{j_a}(\{a\})$. We conclude that:

$$I(E) - I(E^{I-\{j_a\}}_{\Omega-\{a\}}) \;\geq\; v_{j_a}(\{a\}). \tag{16}$$

Consider now the optimal fractional solution x^i_D $(D \subseteq \Omega,\ i \in I)$ to $\mathbf{LP}(E)$. For any $D \subseteq \Omega - \{a\}$, define $y^i_D = x^i_D + x^i_{D\cup\{a\}}$. The variables y^i_D, for $D \subseteq \Omega - \{a\}$ and $i \in I$ provide a feasible fractional solution to $\mathbf{LP}(E^{I}_{\Omega-\{a\}})$. Let M be the value of this solution. We have:

$$L(E^{I}_{\Omega-\{a\}}) \;\geq\; M \tag{17}$$

Let us now bound M from below, making use of the fact that valuations are complement-free.

$$M = \sum_{i\in I} \sum_{D\subseteq\Omega-\{a\}} y^i_D\, v_i(D) = \sum_{i\in I} \sum_{D\subseteq\Omega-\{a\}} x^i_D\, v_i(D) + \sum_{i\in I} \sum_{D\subseteq\Omega-\{a\}} x^i_{D\cup\{a\}}\, v_i(D)$$

Therefore,

$$M - L(E) = \sum_{i \in I} \sum_{D \subseteq \Omega - \{a\}} x^i_{D \cup \{a\}} \left(v_i(D) - v_i(D \cup \{a\})\right)$$

and

$$L(E) - M = \sum_{i \in I} \sum_{D \subseteq \Omega - \{a\}} x^i_{D \cup \{a\}} \left(v_i(D \cup \{a\}) - v_i(D)\right).$$

Since the valuations v_i are complement-free:

$$v_i(D \cup \{a\}) \;\leq\; v_i(D) + v_i(\{a\})$$

and by (15)

$$v_i(D \cup \{a\}) - v_i(D) \;\leq\; v_{j_a}(\{a\}).$$

Therefore

$$L(E) - M \;\leq\; v_{j_a}(\{a\}) \left(\sum_{i \in I} \sum_{D \subseteq \Omega - \{a\}} x^i_{D \cup \{a\}} \right) = v_{j_a}(\{a\}) \left(\sum_{i \in I} \sum_{D \subseteq \Omega, a \in D} x^i_D \right).$$

By constraint (8), then:

$$L(E) - M \;\leq\; v_{j_a}(\{a\})$$

and by (17) and (16)

$$L(E) - L(E^I_{\Omega - \{a\}}) \;\leq\; v_{j_a}(\{a\}) \;\leq\; I(E) - I(E^{I - \{j_a\}}_{\Omega - \{a\}}).$$

∎

7 Existence of a competitive equilibrium

We recall Bikhchandani and Mamer's [4] result.

Theorem 2 *The economy E possesses a competitive equilibrium iff $I(E) = L(E)$.*

We may now state and prove our main result.

Theorem 3 *If E is a complement-free fully competitive economy, then E possesses a competitive equilibrium.*

Proof: The proof proceeds by induction on the number of items $m = \mid \Omega \mid$ of the economy E. The base case is $m = 1$. Any such economy has a competitive equilibrium. Assume the claim has been proved for economies with at most m items. Assume E is a complement-free fully competitive economy with $\mid \Omega \mid = m + 1$. Since E is complement-free, by Lemma 1, there is a trader $j \in I$ and an item $a \in \Omega$ such that

$$I(E) - I(E^{I-\{j\}}_{\Omega-\{a\}}) \ \geq \ L(E) - L(E^{I}_{\Omega-\{a\}}).$$

Since E is fully competitive, the sub-economy $E^{I-\{j\}}_{\Omega-\{a\}}$ is also fully competitive (and complement-free) and by the induction hypothesis, it possesses a competitive equilibrium, and, by Theorem 2:

$$I(E^{I-\{j\}}_{\Omega-\{a\}}) = L(E^{I-\{j\}}_{\Omega-\{a\}}).$$

We have

$$I(E) \ \geq \ L(E) + L(E^{I-\{j\}}_{\Omega-\{a\}}) - L(E^{I}_{\Omega-\{a\}}).$$

But E is fully competitive and the economy $E^{I}_{\Omega-\{a\}}$ is therefore competitive and

$$L(E^{I-\{j\}}_{\Omega-\{a\}}) = L(E^{I}_{\Omega-\{a\}}).$$

We conclude that $I(E) = L(E)$ and, by Theorem 2, E possesses a competitive equilibrium. ∎

8 Economies of size 2

For economies with 2 or less items, the restriction to complement-free valuations is not needed.

Theorem 4 *If $E = \langle \Omega; I; v_i : i \in I \rangle$ is a fully competitive economy with two items: $\mid \Omega \mid = 2$, then (*without any assumption of complement-freeness*) it possesses a competitive equilibrium.*

Proof: We shall prove the result directly, i.e., without using Bikhchandani and Mamer's result. Let $\Omega = \{a, b\}$. Since E is fully competitive, $E^{I}_{\{a\}}$ is competitive and there are at least two agents k_1 and k_2, $k_1 \neq k_2$ that give maximal value to a: $v_{k_1}(\{a\}) = v_{k_2}(\{a\}) \geq v_i(\{a\})$ for any $i \in I$. In fact, since $E_{\{a\}}^{I-k_1}$ is also competitive, there are even three pairwise distinct agents for which the maximal value is taken and similarly for b.

We conclude that there are agents l_a and l_b, $l_a \neq l_b$ such that $v_{l_a}(\{a\}) \geq v_i(\{a\})$ and $v_{l_b}(\{b\}) \geq v_i(\{b\})$ for any $i \in I$.

We now distinguish two cases. First, if allocating a to l_a and b to l_b provides an optimal integral allocation, then, the prices $p_a = v_{l_a}$ and $p_b = v_{l_b}$ are competitive since for every $i \in I$: $v_i(\{a,b\}) \leq p_a + p_b$, $v_i(\{a\}) \leq p_a$ and $v_i(\{b\}) \leq p_b$, whereas $v_{l_a}(\{a\}) = p_a$ and $v_{l_b}(\{b\}) = p_b$.

Then, otherwise, the optimal allocation allocates both a and b to the same agent j and we have $v_j(\{a,b\}) \geq v_i(\{a,b\})$ for any $i \in I$, and also $v_j(\{a,b\}) \geq v_{l_a}(\{a\}) + v_{l_b}(\{b\})$. One may find prices p_a and p_b such that: $p_a + p_b = v_j(\{a,b\})$, $p_a \geq v_{l_a}(\{a\})$ and $p_b \geq v_{l_b}(\{b\})$. They define a competitive equilibrium. ∎

9 Negative example

The following example shows that, in general, the no-complementarity assumption in Theorem 3 cannot be dispensed with.

Example 5 *The economy E_2 of Example 2 is fully competitive as noticed in Example 4 but possesses no competitive equilibrium. This last fact follows from Theorem 2 and the fact that $L(E_2) = 15$ whereas $I(E_2) = 11$. Equivalently, a direct inspection shows that, since the allocation part of a competitive equilibrium is an optimal allocation, it must allocate two of the items (say a and b) to some agent and the third item (c) to some other agent. Equilibrium prices must be such that the sum of the prices of a and b is at least* 10 *(since there are agents that get nothing), that both p_a and p_b must be at most p_c (since otherwise the agent who gets a and b would prefer c to one of them) and the price of c, p_c must be at most* 1 *(since the agent who gets c prefers this to the empty bundle). There are no such prices. Notice that an infinity or even a continuum of traders as in [2, 3] does not guarantee a competitive equilibrium.*

10 Conclusions and Further Work

In an exchange of indivisible objects the notion of a competitive market, i.e., a market in which no trader can influence (downwards) the prices is much easier to define that in the case of divisible objects. There is no need, in particular, to consider an infinity of traders. The definition of perfect competition that we proposed ensures the existence of a competitive equilibrium only among traders that exhibit no complementarity. The following questions are open. Is there a stronger notion of perfect competition that ensures the existence of such equilibria for a larger class of traders? Does any complement-free and competitive (not fully competitive) economy possesses a competitive equilibrium? What can one say about economies E in which the gap between $L(E)$ and $I(E)$ is non-zero but small?

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